Per-Antenna-Coded Schemes for MIMO OFDM

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***Abstract*—Two Per-Antenna-Coding (PAC) receiver schemes for Multiple-Input Multiple-Output (MIMO) Orthogonal Frequency Division Multiplexing (OFDM) are described and compared in this paper. In MIMO, the data to transmit are multiplexed onto different antennas. With PAC (sometimes called horizontal coding), these different data streams are encoded separately. For the receiver, two schemes are proposed to detect and decode the PAC transmission. One is based on Soft-decision Output Maximum Likelihood Detection (SOMLD), the other on V-BLAST. When forward error correcting decoding is included in the Successive Interference Cancellation loops of V-BLAST, its performance is shown to be comparable with that of SOMLD.**

Keywords—MIMO systems; Orthogonal Frequency Division Multiplexing (OFDM); Space Division Multiplexing; Wireless LAN.

I. INTRODUCTION

Applying multiple antennas at both the transmitter and receiver side can, especially when the environment provides rich scattering, greatly improve the capacity/throughput of a wireless communication link in flat-fading [1], as well as frequency-selective fading channels [2]. To deal with the frequency-selective nature of broadband wireless channels, Orthogonal Frequency Division Multiplexing (OFDM) can be combined with multiple-transmit multiple-receive antenna (i.e., Multiple-Input Multiple-Output (MIMO)) techniques. OFDM transforms a frequency-selective channel into parallel flat-fading channels. In this way, MIMO algorithms, which usually are designed for flat-fading, can be applied in broadband communication [3]. Another nice property of OFDM is its robustness against Inter-Symbol Interference (ISI).

Spatial multiplexing has been recognized as one of the MIMO techniques to exploit (most of) the available capacity [4-6]. In [4] it has been shown that Maximum Likelihood Detection (MLD) is the best performing scheme, followed by V-BLAST [5]. To achieve more robustness, extra coding can be applied on top of the spatial multiplexing, see e.g. [6]. The main disadvantage of MLD is the fact that its complexity grows exponentially with the number of transmit antennas N_t. Therefore, research to find well-performing, less complex schemes is ongoing.

In this paper, we introduce a well-performing scheme with a polynomial complexity in N_t. The scheme is an extension to V-BLAST and based on Per-Antenna-Coding at the transmitter. Instead of first doing the Successive Interference Cancellation (SIC) ([5]) and then the decoding, the forward error correcting decoding is included in the SIC loops of V-BLAST. The performance of this scheme is further enhanced by letting it generate soft-decision output values. This so called PAC V-BLAST scheme is compared to PAC Soft-Decision Output MLD (SOMLD) and the performance is shown to be similar.

II. EQUIVALENT BASEBAND SIGNAL MODEL

Consider a communication system with N_t transmit (TX) and N_r receive (RX) antennas. Assume that the system is operating in a frequency-selective Rayleigh fading environment and that the channel coefficients remain constant during a packet transmission, i.e., quasi-static fading. And, furthermore, suppose that the channel impulse response can be recorded with L samples. Then, the fading channel between the p-th TX and q-th RX antenna can be modeled by a discrete-time baseband equivalent (L−1)-th order finite impulse response (FIR) filter with filter taps $g_{qp}[l]$ ($l = \{0, \dots, L-1\}$). We assume that these taps are independent zero-mean complex Gaussian random variables with variance $0.5P[l]$ per dimension. The ensemble $P[l]$, $l = \{0, \dots, L-1\}$, is called the Power Delay Profile (PDP), and is assumed to be normalized to $\sigma_c^2 = 1$ (i.e., the average channel or propagation attenuation).

To deal with the frequency-selectivity of the channel, we apply OFDM utilizing N_c subcarriers per antenna transmission. Since we assume quasi-static fading we will omit time indices. If we denote the $N_t \times 1$ MIMO vector that is transmitted on the i-th subcarrier by $\mathbf{s}[i]$, then, after FFT processing, the received baseband vector can be expressed as

$$\mathbf{x}[i] = \mathbf{H}[i] \cdot \mathbf{s}[i] + \mathbf{n}[i], \qquad (1)$$

where the $N_r \times 1$ vector $\mathbf{n}[i]$ represents Additive White Gaussian Noise (AWGN) on the i-th subcarrier with independent and identically distributed (i.i.d.) zero-mean, complex Gaussian elements with variance σ_n^2. Let $g_{qp}[l]$ denote the (q,p)-th element of $\mathbf{G}[l]$, then (assuming that there is no Inter Symbol Interference (ISI))

$$\mathbf{H}[i] = \sum_{l=0}^{L-1} \mathbf{G}[l] \cdot e^{-j2\pi li/N_c} \; . \qquad (2)$$

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III. SYSTEM DESCRIPTION

In case of MIMO, encoding can be done either jointly over the multiple transmitter branches, or per branch ([6]). The latter option is the topic of this paper and is called Per-Antenna-Coding (PAC). A transmitter scheme, in which PAC is applied to MIMO OFDM, is given in Fig. 1. Basically the MIMO OFDM transmitter consists of N_t OFDM transmitters among which the incoming bits are spread, then each branch in parallel performs encoding, interleaving (Π), QAM mapping, N_c-point Inverse Fast Fourier Transformation (IFFT), and adds the cyclic extension before the final TX signal is upconverted to RF and sent.

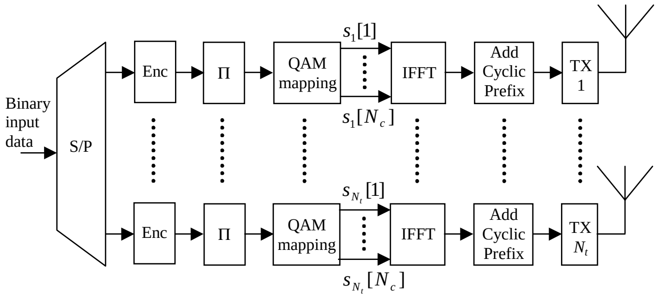

Fig. 1: PAC MIMO OFDM transmitter scheme.

Since the MIMO algorithms that will be compared in this paper are single carrier algorithms, in combination with OFDM, at the receiver, a MIMO detection algorithm has to be performed per subcarrier. For a $N_t \times N_r$ system, every subcarrier bears N_t data streams. At the N_r receivers, the subcarrier information is separated by performing the N_c-point Fast Fourier Transformation (FFT) (see Fig. 2). Then, in general, the symbols mapped onto subcarrier i are routed to the i-th MIMO detector to recover the N_t transmitted data signals per subcarrier ([3]). Finally, demapping, deinterleaving (Π^{-1}) and decoding are performed per receiver branch and the resulting data are combined to obtain the binary output data. In the next sections, it is explained how the Detection and Decoding Block of Fig. 2 is filled in for Soft-Decision Output MLD and Per-Antenna-Coded V-BLAST.

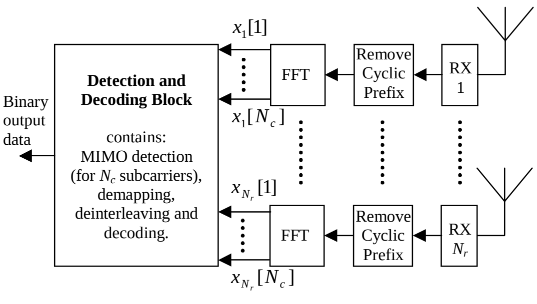

Fig. 2: PAC MIMO OFDM receiver scheme.

A. *Soft-Decision Output Maximum Likelihood Detection*

In the case of Soft-Decision Output MLD[1] (SOMLD), following [3], the Detection and Decoding Block of Fig. 2 consists of N_c ML detectors, which produce soft estimates (i.e., soft-decision output values) of the coded bits transmitted on the respective subcarrier. To find these soft-decision outputs, we can use the same approach as [7], where the Log Likelihood Ratio (LLR) is used as an indication for the reliability of a bit. Suppose that, at a given time instance, $K = N_t \cdot m$ bits are sent on a certain subcarrier, where $m = \log_2 M$ denotes the amount of bits used per M-QAM constellation point. Then (omitting the subcarrier index), if b_k is the k-th bit of the transmitted vector to estimate, the LLR for this bit is

$$L(b_k) = \ln \frac{\Pr(b_k = +1|\mathbf{x})}{\Pr(b_k = -1|\mathbf{x})} = \ln \frac{\sum_{\mathbf{s}_j|b_k=+1} \Pr(\mathbf{s}_j|\mathbf{x})}{\sum_{\mathbf{s}_j|b_k=-1} \Pr(\mathbf{s}_j|\mathbf{x})}, \tag{3}$$

where the ensemble $\mathbf{s}_j$ ($1 \le j \le J$) denotes all possible transmitted vectors on a certain subcarrier, thus,

$$J = M^{N_t}. \tag{4}$$

When we apply Bayes' rule: $\Pr(A|B) = \Pr(B|A) \cdot \Pr(A)/\Pr(B)$, the LLR becomes

$$L(b_k) = \ln \frac{\sum_{\mathbf{s}_j|b_k=+1} p(\mathbf{x}|\mathbf{s}_j)\Pr(\mathbf{s}_j)}{\sum_{\mathbf{s}_j|b_k=-1} p(\mathbf{x}|\mathbf{s}_j)\Pr(\mathbf{s}_j)}. \tag{5}$$

Because the vectors $\mathbf{s}_j$ are equally likely to be transmitted, $\Pr(\mathbf{s}_j)$ is equal for all vectors $\mathbf{s}_j$. Since we assume that the vector $\mathbf{x}$ is the result of a MIMO transmission over a flat-fading Rayleigh channel (i.e., each subcarrier is treated flat over frequency), we know that this vector $\mathbf{x}$ has a complex multivariate normal distribution [3]. So, for a given channel matrix $\mathbf{H}$, the conditional probability density function can be shown to be

$$\begin{aligned} p(\mathbf{x}|\mathbf{H},\mathbf{s}_j) &= \\ &\det(\pi\mathbf{Q})^{-1} \exp\left(-(\mathbf{x}-\mathbf{H}\mathbf{s}_j)^H \mathbf{Q}^{-1}(\mathbf{x}-\mathbf{H}\mathbf{s}_j)\right), \end{aligned} \tag{6}$$

where $\mathbf{Q}$ is the covariance matrix and equals

$$\begin{aligned} \mathbf{Q} &= E[(\mathbf{x}-\boldsymbol{\mu})(\mathbf{x}-\boldsymbol{\mu})^H] \\ &= E\left[(\mathbf{x}-\mathbf{H}\mathbf{s}_j)(\mathbf{x}-\mathbf{H}\mathbf{s}_j)^H\right] = E\left[\mathbf{n}\mathbf{n}^H\right] = \sigma_n^2 \mathbf{I}_{N_r}. \end{aligned} \tag{7}$$

In this result $\mathbf{I}_y$ represents the $y \times y$ identity matrix. Finally, we arrive at the LLR of b_k

[1] Also called maximum a posteriori probability (MAP) detection.

$$L(b_k) = \ln \frac{\sum_{\mathbf{s}_j | b_k = +1} \exp\left(- \frac{\left\| \mathbf{x} - \mathbf{H}\mathbf{s}_j \right\|^2}{\sigma_n^2} \right)}{\sum_{\mathbf{s}_j | b_k = -1} \exp\left(- \frac{\left\| \mathbf{x} - \mathbf{H}\mathbf{s}_j \right\|^2}{\sigma_n^2} \right)}. \quad (8)$$

When applying the so-called max-log approximation this results in

$$L(b_k) \approx \frac{1}{\sigma_n^2} \left(\min_{\mathbf{s}_j | b_k = -1} \left\| \mathbf{x} - \mathbf{H}\mathbf{s}_j \right\|^2 - \min_{\mathbf{s}_j | b_k = +1} \left\| \mathbf{x} - \mathbf{H}\mathbf{s}_j \right\|^2 \right). \quad (9)$$

Note that this type of decoding involves an exhaustive search over all possible vectors $\mathbf{s}_j$, leading to a complexity that grows exponentially with N_t.

B. Per-Antenna-Coded V-BLAST

The fact that the complexity of MLD grows exponentially with the number of transmit antennas resulted in the research to less complex algorithms. One of the potential schemes is called V-BLAST [5, 8] and is based on Minimum-Mean-Square-Error (MMSE) detection with Decision-Feedback Equalization (DFE). The complexity of this scheme grows polynomial with N_t [8] and when using the proposed Optimal Successive Interference Cancellation (OSIC) in [5], the performance degradation with respect to MLD is small compared to linear techniques like Zero-Forcing [4].

In this paper, we propose an extension to V-BLAST. It makes use of the Per-Antenna-Coding at the transmitter and we will call it PAC V-BLAST. The idea is to first go through the decoding stage before the Successive Interference Cancellation (SIC) is executed. In this way Forward Error Correcting coding is perform on the SIC information. How this can be applied to MIMO OFDM is schematically represented in Fig. 3.

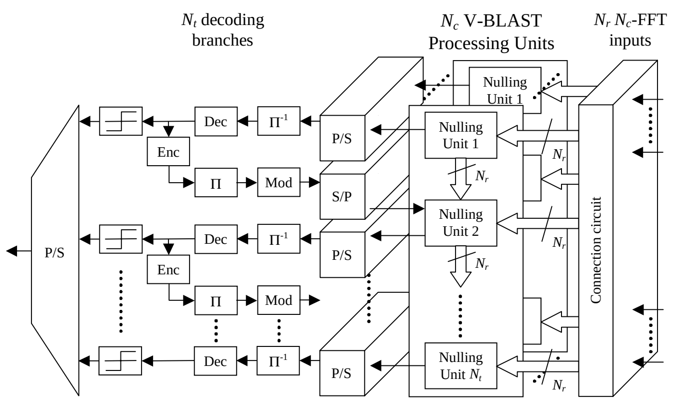

Fig. 3: The PAC V-BLAST Detection and Decoding Block.

To improve the performance even more, the V-BLAST detection should produce soft-decision outputs. In order to obtain them, the estimated values of each Nulling Unit should not be sliced to their respective QAM points, as done in [5], but they should go through a Single-Input Single-Output (SISO) implementation, i.e. the 1×1 version, of the SOMLD described in the previous section.

Note that, since the SIC information is passed through a decoder and encoder stage, a disadvantage of this scheme could be its latency. But when the interleaver is not too big, and convolutional encoding and Viterbi decoding is used, the encoder in the SIC feedback loop can start its operation as soon as the Viterbi decoder produces outputs. Then, the latency is manageable.

IV. Performance Comparison

A. SNR Definition

In the baseband processing of a MIMO OFDM transmission system, there are a number of subsequent blocks that have an influence on the relation between bit-energy to noise ratio E_b/N_0 and the symbol-energy to noise ratio E_s/N_0 (both per RX antenna). These blocks are:

- the encoder with coding rate R ($R < 1$),
- the modulation block that maps m bits on a 2^m-ary modulation scheme,
- the spatial mapper that maps N_t symbols on N_t transmit antennas,
- the N_c-point IFFT that maps N_n symbols on N_n sub-carriers ($N_n < N_c$) ,
- and a block that adds the cyclic prefix, by adding a guard interval of T_G to the OFDM symbol length T, leading to a total symbol length of $T_{\text{tot}} = T_G + T$.

Now, assume that the communication between transmitter and receiver is scaled such that the variance of the propagation attenuation equals $\sigma_c^2 = 1$. Then, the ratio between the SNR per receive antenna (E_s/N_0) and E_b/N_0 for a MIMO OFDM system is given by

$$\frac{E_s}{N_0} = \frac{T_s}{T_b}\frac{E_b}{N_0} = N_t R m \frac{N_n T}{N_c T_{\text{tot}}}\frac{E_b}{N_0} = \eta_{\text{eff}} \frac{E_b}{N_0}. \quad (10)$$

Note that E_s/N_0 is equal to E_b/N_0 multiplied by the ratio of the bit rate and bandwidth. The latter ratio is equivalent to the spectral efficiency η_{eff} in bps/Hz.

B. Simulation Results

A number of simulations are performed to compare the Bit-Error Rate (BER) performance of the two Per-Antenna-Coding detection schemes described in Section III. The main simulation parameters are based on the OFDM standard IEEE 802.11a [9]. To obtain our results, we averaged over 50,000 2*54 byte packets. Assuming quasi-static fading, every packet is sent through a channel realization, of which the elements

$g_{qp}[l]$ are modeled according to Section II with an exponential decaying PDP (i.e., the ensemble $P[l]$ falls of exponentially) with an rms delay spread of 0, 50 and 250 ns. Following the IEEE 802.11a standard, a cyclic extension (i.e., a guard interval) of 800 ns is used. This interval is introduced to provide robustness to delay spreads up to several hundreds of nanoseconds. In practice, this means that the system is robust enough to be used in any indoor environment [9].

Furthermore, N_n = 48 out of N_c = 64 data subcarriers are used for data transmission and the modulation that is applied is Binary Phase Shift Keying (BPSK). In order to correct for subcarriers in deep fades, a forward error correcting code across the subcarriers is used, namely, rate ½ convolutional coding with constraint length 7 and generator polynomials (133,171). Finally, perfect Channel State Information and perfect synchronization is assumed at the receiver.

With these parameters, the BER performances versus the E_b/N_0 per receive antenna is depicted for PAC SOMLD and V-BLAST, for different delay spreads and for a 2×2 and 4×4 antenna setup, in Fig. 4 and Fig. 5, respectively.

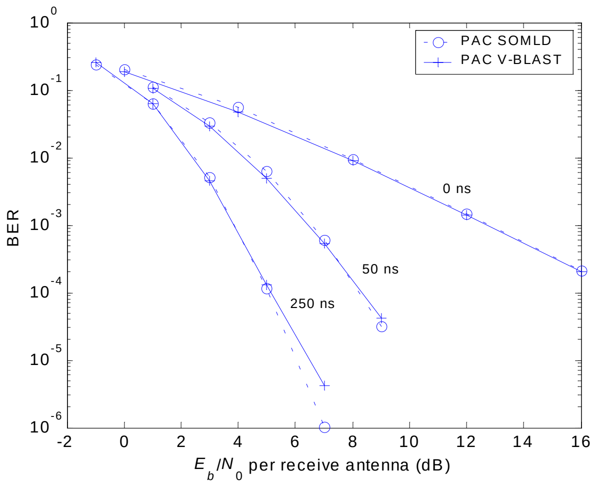

Fig. 4: BER performance for 2×2 Per-Antenna-Coding schemes, BPSK, rate ½ coding and frequency-selective Rayleigh fading with different rms delay spreads.

From these results, we can conclude that, for low SNR values, the performance of PAC V-BLAST is better than that of PAC SOMLD. For higher SNRs, the performance of PAC V-BLAST deteriorates compared to PAC SOMLD. The latter can be explained most likely by the fact that the diversity order of V-BLAST finally tends towards $N_r - N_t + 1$, whereas that of SOMLD is equal to N_r [4]. The former can be explained by the way the soft-decision output values are generated. Due to non-orthogonal channels, received MIMO vectors will have dependent elements, which again will result in dependent soft-values for SOMLD as defined by (3). It is well known that the Viterbi decoder only performs optimally if the input values are independent. A solution would be to calculate joint LLRs ([10]) and, accordingly, change the decoder to handle these joint soft-values. Since V-BLAST is based on the MMSE algorithm, it first orthogonalizes the data streams and then determines the soft-values, so V-BLAST does not have the above mentioned problem, which can explain its better performance.

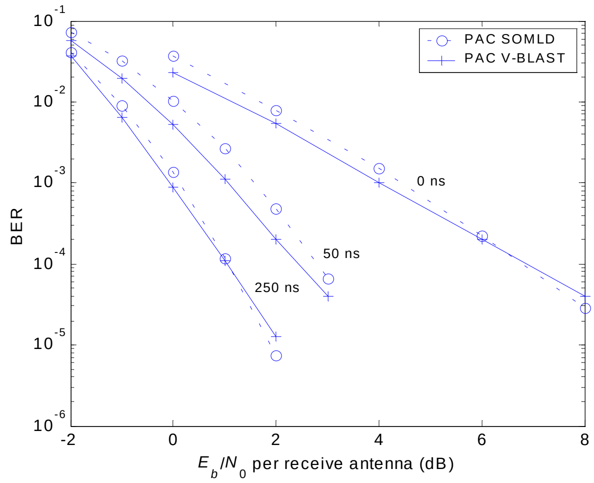

Fig. 5: BER performance for 4×4 Per-Antenna-Coding schemes, BPSK, rate ½ coding and frequency-selective Rayleigh fading with different rms delay spreads.

V. Complexity Analysis

In this section, the complexity of PAC SOMLD and PAC V-BLAST will be compared. In this comparison, we will only compute the complexity of the MIMO detectors, since the rest of the transmission scheme is the same for both detection techniques.

A. Complexity of PAC SOMLD

For simplicity we will assume that the complexity of SOMLD is very similar to hard-decision output MLD. In case of ML detection, prior to detection (in the training phase) we can already calculate $\mathbf{Hs}_j$, for $1 \le j \le J$, using the fact that

$$\mathbf{Hs}_j = \sum_{i=1}^{N_t} \mathbf{H}_i (s_i)_j , \quad (11)$$

where $\mathbf{H}_i$ denotes the i-th column of $\mathbf{H}$ and $(s_i)_j$ the i-th element of vector $\mathbf{s}_j$. So, e.g. for a 2×2 system with BPSK, we need to calculate $\mathbf{H}_1\cdot(+1)$, $\mathbf{H}_1\cdot(-1)$, $\mathbf{H}_2\cdot(+1)$ and $\mathbf{H}_2\cdot(-1)$, and sum all possible combinations. Suppose N_n out of N_c subcarriers are used. Then, the complexity to calculate $\mathbf{H}_i\cdot(s_i)_j$, taking all M possibilities of $(s_i)_j$ into account, would be MN_tN_r (Flops) per subcarrier, or $N_nMN_tN_r$ (Flops) in total. Summing all possible combinations has a complexity of

$$N_n \sum_{i=2}^{N_t} M^i N_r = N_n M^2 N_r \frac{M^{N_t - 1} - 1}{M - 1} \text{ (Flops).} \quad (12)$$

In the data phase, where the actually detection of the payload is performed, per received MIMO vector, the subtraction $\mathbf{x} - \mathbf{Hs}_j$ needs to be done for the J vectors $\mathbf{s}_j$ (complexity: $J\cdot N_r$ (Flops)). From these results the norms have to be calculated (i.e., $(2N_r - 1)\cdot J$ (Flops)) and the minimum needs to be found ($J-1$ (Flops)). In total, the ML detection complexity for a payload of N_s MIMO OFDM symbols is about

$$3N_n N_s N_r M^{N_t} \text{ (Flops).} \quad (13)$$

B. Complexity of PAC V-BLAST

In [8], the complexity of an efficient single-carrier implementation of V-BLAST per packet transmission is given. Here, we will extend this to the multi-carrier case. For V-BLAST, per used subcarrier, the weight vectors to perform the nulling and optimal ordering need to be determined (this can be done in the training phase). In the multicarrier case, this would require N_n times the complexity given in [8], thus

$$N_n\left(\tfrac{2}{3}N_t^3 + 7N_t^2 N_r + 2N_t N_r^2\right) \text{ (Flops).} \quad (14)$$

The complexity of detecting the payload is given by [8]

$$2N_n N_s N_t^2 \text{ (Flops).} \quad (15)$$

Finally, different from [8], we also need to calculate the soft-decisions of the coded bits. This would lead in the training phase to a complexity of (see Section III.A with $N_t = N_r = 1$) $N_n \cdot M$ (Flops) and to a complexity of $3N_n \cdot N_s \cdot M$ (Flops) in the data phase.

C. Complexity Comparison

Now, the complexity between PAC SOMLD and PAC V-BLAST can be compared. For a MIMO OFDM system with QPSK or 16-QAM, rate ½ coding, a packet length of 2×54 bytes, and in which $N_n = 48$ subcarriers are used for data transmission, the total complexity as a function of $N_t = N_r$, is given in Fig. 6. Since, for a larger packet length, the payload-detection complexity of PAC SOMLD increases more than that of PAC V-BLAST, in general it can be concluded that the complexity of PAC V-BLAST is lower for $N_t = N_r \geq 2$.

VI. Conclusions

Two Per-Antenna-Coding receiver schemes for MIMO OFDM have been described and compared in this paper. One is Soft-decision Output MLD, the other is PAC V-BLAST. The performance of the latter is similar to that of the former, whereas its complexity grows only polynomial with the number of transmit antennas, instead of exponentially. Similar performance is achieved at the expense of latency. For applications in quasi-static fading, it can be shown, however, that the latency is manageable.

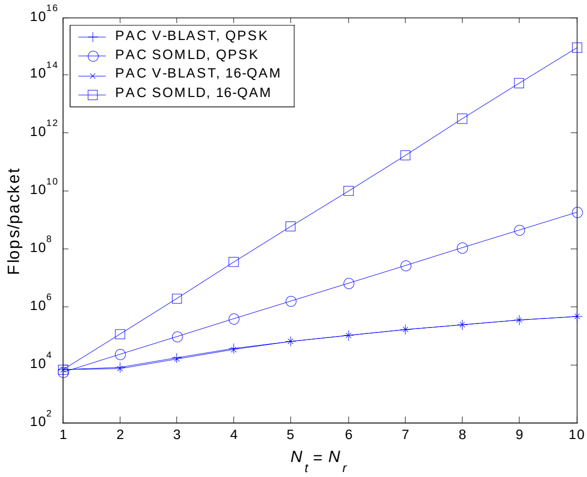

Fig. 6: Flops of the training + data phase complexity of PAC VBLAST and PAC SOMLD, for QPSK or 16-QAM and a 2×54 byte packet vs $N_t = N_r$.

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