Space Division Multiplexing Algorithms

A. van Zelst, *Student Member, IEEE*

Abstract— The main goals in developing new wireless communication systems are increasing the transmission capacity and improving the spectrum efficiency. In this paper, spectrum efficient and capacity improving techniques, captured under the name Space Division Multiplexing (SDM), are proposed. SDM algorithms exploit the richly scattered (indoor) wireless channel by transmitting different signals simultaneously on different transmit antennas at the same frequency and using multiple receive antennas for decoding. Among these algorithms, Maximum Likelihood Decoding (MLD) is shown to have the best SNR performance.

Index Terms— Space Division Multiplexing, Multi Input Multi Output Channel, Maximum Likelihood Decoding.

I. Introduction

Due to the rapid growth of the amount of information transported over communication systems and the demand for mobility, a lot of research into new broadband wireless communication systems is ongoing. Large-scale penetration of such systems into our daily lives will require increases in bit rate (system capacity) and spectrum efficiency.

A promising solution is the exploitation of the spatial dimension, since recent information theory research has revealed that a richly scattered multipath wireless channel (e.g., like indoor environments) is capable of enormous capacities ([1]-[3]). The multipath can be exploited properly by transmitting different data streams on different transmit antennas simultaneously. Although, these different data streams are mixed-up in the air, they can be recovered when multiple receive antennas are used as well. The principle behind this, is that, if the antennas are spaced far enough apart (i.e., at least half a wavelength [4]), the richly scattered multipath creates more or less independent paths between the different transmit and receive antennas.

The decoding techniques described in this paper can be captured under the term Space Division Multiplexing (SDM) or Space Division Multiple Access (SDMA). The difference between SDM and SDMA is that the latter allows different users to transmit simultaneously on a single antenna each, whereas in SDM a single user transmits simultaneously on multiple antennas. Hybrid schemes where several users transmit simultaneously on different antennas may also be possible.

The Bell Laboratories Layered Space-Time (BLAST) technique ([5]), is such an SDM approach. With a prototype of BLAST, bandwidth efficiencies of 20 – 40 bits per second per Hertz (bps/Hz) have been demonstrated in an indoor environment at realistic SNRs and error rates.

Closely related to the SDM techniques are the so-called Space-Time codes ([6],[7]). The difference is that for Space-Time codes, the data is coded in the space (on different antennas) and time dimension to maximize the coding gain and diversity gain. Thus, one data-symbol is coded and all antennas are used to transmit it. SDM achieves higher bit rates by transmitting different data-symbols on the different transmit antennas simultaneously. Coding gain can be obtained by encoding the data in advance, e.g., with a convolutional code.

In this paper, a number of SDM algorithms will be described and their SNR performance will be compared, but first a signal model for the Multi Input Multi Output channel will be stated.

II. Multi Input Multi Output Channel Model

In this section, a signal model for the Multi Input Multi Output (MIMO) channel is given. The communication channel bandwidth is assumed to be so narrow that the channel can be treated as flat with frequency (i.e., flat fading).

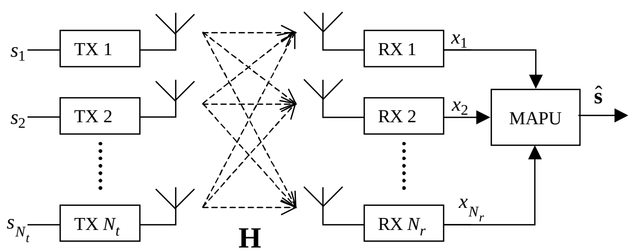

Figure 1: The physical model of a system with SDM. (MAPU = Multi Antenna Processing Unit)

A single-user link with N_t transmit (TX) and N_r receive (RX) antennas is considered. It is assumed to operate in a Rayleigh flat-fading environment and exploits the spatial dimension by using Space Division Multiplexing (SDM) (see Figure 1). At discrete times, the transmitter sends an N_t-dimensional (complex) signal vector $\mathbf{s}$. The receiver

A. van Zelst is a Ph.D. student at the Eindhoven University of Technologies, Eindhoven, The Netherlands and Bell Labs, Lucent Technologies, Nieuwegein, The Netherlands, email: zelst@lucent.com.

records an N_r-dimensional complex vector **x**. The following signal model describes the relation between **s** and **x**:

$$\mathbf{x} = \mathbf{Hs} + \mathbf{v} \tag{1}$$

where **H** is an $N_r \times N_t$ complex propagation matrix that is constant with respect to the symbol time and only assumed known at the receiver (e.g. via transmitting training sequences). Since it is assumed that the system operates in a Rayleigh flat-fading environment, it can be said that **H** has independently and identically distributed (i.i.d.), zero-mean, circularly-symmetric, complex Gaussian entries with unit variance (the variance of each entry is $\sigma_c^2 = 1$) [8]. The N_r-dimensional vector **v** represents zero mean, complex Additive White Gaussian Noise (AWGN) with covariance matrix:

$$E[\mathbf{v}\mathbf{v}^*] = \sigma_v^2 \mathbf{I}_{N_r} \tag{2}$$

where * denotes the conjugate transpose of a vector or matrix. The matrix **I** with subscript N_r represents the identity matrix with dimension N_r. The vector **s** is assumed to have zero-mean, uncorrelated random variables with variance σ_s^2. The total power of **s** (i.e., $E[\mathbf{s}^*\mathbf{s}]$) is assumed to be P (independent of the number of transmit antennas!). Thus, the covariance matrix of **s** equals:

$$E\left[\mathbf{s}\mathbf{s}^*\right] = \sigma_s^2 \mathbf{I}_{N_t} = \frac{P}{N_t} \mathbf{I}_{N_t} \tag{3}$$

Furthermore, the vectors **s** and **v** are assumed to be independent ($E[\mathbf{s}\,\mathbf{v}^*]=0$). Now, the expected Signal-to-Noise Ratio (SNR) per receiving antenna, i.e. the SNR for each component of **x**, can be found and is equal to:

$$\rho = \frac{E_s}{N_0} = \frac{N_t \sigma_s^2 \sigma_c^2}{\sigma_v^2} = \frac{P}{\sigma_v^2} \tag{4}$$

where E_s stands for the signal power per receive antenna and N_0 denotes the noise power per receive antenna.

III. Capacity

Provided that the channel matrix **H** is known at the receiver, the Shannon capacity for an SDM system is given by ([1]):

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^*\right) \text{ bps/Hz} \tag{5}$$

As an example, the capacity for $N_r=N_t=N$ in the limit of large N is considered: by the law of large numbers, $\mathbf{HH}^*/N \rightarrow \mathbf{I}_N$ almost surely as N gets large, so, the capacity for large N is asymptotic to:

$$C = N \log_2(1+\rho) \text{ bps/Hz} \tag{6}$$

From this formula, it can be concluded that for high SNRs and $N_r=N_t=N$, the scaling of the capacity is like N more bps/Hz for every 3 dB SNR improvement, whereas, for a conventional wireless communication system ($N_t=N_r=1$) with the same total transmission power P, the scaling is only 1 bps/Hz [1].

IV. Space Division Multiplexing Algorithms

A. The Zero Forcing Algorithm

The first decoding technique to be described in this paper is the so-called Zero Forcing (ZF) algorithm. With ZF, the estimates of the transmitted vector ($\mathbf{s}_{\text{est}}$) are obtained at the receiver, using the following processing:

$$\mathbf{s}_{\text{est}} = \left(\mathbf{H}^*\mathbf{H}\right)^{-1}\mathbf{H}^*\mathbf{x} = \mathbf{H}^+\mathbf{x} \tag{7}$$

where $^+$ represents the *pseudo-inverse*. In order for the pseudo-inverse to exist, N_t must be less than or equal to N_r, otherwise $\mathbf{H}^*\mathbf{H}$ is singular and its inverse does not exist [9]. Furthermore, note that the inverse only exists if the columns of **H** are independent, which is the case, since it is assumed that the elements of **H** are i.i.d.

As a final step, $(s_{\text{est}})_i$, i.e. the i-th element of $\mathbf{s}_{\text{est}}$, can be sliced to the nearest Quadrature Amplitude Modulation (QAM) constellation point. These sliced signals are denoted by $\hat{\mathbf{s}}$. In this way, all N_t elements of **s** can be decoded at the receiver.

The ZF algorithm can be simulated and its Bit Error Rate (BER) performance can be obtained (see Section V). It is possible to verify these simulations when the fact is used that the diversity order[1] of the average error probability of ZF with antenna configuration (N_t,N_r), is equal to a system with Maximal Ratio Combining (MRC), one TX antenna and $N_r–N_t+1$ RX antennas [3]. According to the diversity analysis of an MRC system with Binary Phase Shift Keying (BPSK) modulation and L receive antennas, performed in [8], the BER can be expressed as:

$$P_{\text{b}} = \left[\frac{1}{2}(1-\mu)\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \cdot \left[\frac{1}{2}(1+\mu)\right]^k \tag{8}$$

where, by definition,

$$\mu = \sqrt{\gamma_c / (1+\gamma_c)} \tag{9}$$

and γ_c denotes the average SNR per subchannel (from one TX antenna to one RX antenna). When γ_c satisfies the

[1] A diversity order of i means that the BER decreases 10^i times it the SNR increases by 10 dB.

condition γ_c >> 1, the term $(1 + \mu)/2 \approx 1$, the term $(1 - \mu)/2 \approx 1/4\gamma_c$ and Formula (8) can be approximated as:

$$P_{\rm b} \approx \left(\frac{1}{4\gamma_c}\right)^L \binom{2L-1}{L} \qquad (10)$$

So, it appears that the error rate decreases inversely with the L-th power of the SNR (i.e., an L-th order of diversity). Finally, substituting $L=N_r-N_t+1$ in Formula (8) gives the exact BER for a ZF solution of an (N_t,N_r) system with BPSK. Note that, if ρ represents the SNR per receiving antenna, the SNR per subchannel equals:

$$\gamma_c = \frac{\rho}{N_t} \qquad (11)$$

B. The Minimum Mean Square Error Solution

Another approach in (linear) estimation theory to the problem of estimating a random vector **s** on the basis of observations **x** is to choose a matrix **D** that minimizes the Mean Square Error:

$$\varepsilon^2 = E[(\mathbf{s}-\mathbf{s}_{\rm est})^*(\mathbf{s}-\mathbf{s}_{\rm est})] = [(\mathbf{s}-\mathbf{Dx})^*(\mathbf{s}-\mathbf{Dx})] \qquad (12)$$

From [10], the linear Minimum Mean Square Error (MMSE) solution can be obtained and is given by:

$$\mathbf{s}_{\rm est} = \mathbf{Dx} = (\alpha\mathbf{I} + \mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*\mathbf{x}\ ,\ \alpha > 0 \qquad (13)$$

where α is equal to $\sigma_v^2/\sigma_s^2 = N_t/\rho$. From Formula (13) it becomes clear that the ZF solution corresponds to an MMSE solution with $\alpha = 0$. On the other hand, the MMSE solution can be considered to be a ZF solution with:

$$\mathbf{H} \rightarrow \begin{bmatrix}\mathbf{H} \\ \sqrt{\alpha}\mathbf{I}_{N_t}\end{bmatrix} \text{ and } \mathbf{x} \rightarrow \begin{bmatrix}\mathbf{x} \\ 0\end{bmatrix} \qquad (14)$$

C. Decision Feedback Decoding

The Zero Forcing approach as described before is viable, but as will become clear from the results in Section V, superior performance is obtained if non-linear techniques are used. One can imagine that if somehow first the most reliable element of the transmitted vector **s** could be decoded and used to improve the decoding of the other elements of **s**, the performance may increase. This is called *symbol cancellation* [5] and it exploits the timing synchronism inherent in the system model (the assumption of co-located transmitters makes this completely reasonable). Furthermore, ZF or MMSE is used to perform detection. In other words, symbol cancellation is based on the subtraction of interference from already detected components of **s** from the receiver signal vector **x**. This results in a modified receiver vector in which, effectively, fewer interferers are present. This principle is called Decision Feedback Decoding (DFB). The BLAST algorithm proposed in [5] is based on DFB and the reader is referred to this paper for an algorithmic description of DFB.

D. Maximum Likelihood Decoding

The only decoding method described in this paper that is not based on calculation of matrix inverses is the Maximum Likelihood Decoding (MLD) algorithm. MLD is a method that compares the received signal with all possible transmitted vectors (modified by **H**) and estimates **s** according to the Maximum Likelihood principle. This principle can be formalized by the following formula:

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s}_j \in \{\mathbf{s}_1,\ldots,\mathbf{s}_K\}} \left\|\mathbf{x} - \mathbf{H}\mathbf{s}_j\right\|^2 \qquad (15)$$

Note that this requires an exhaustive search through all possible transmitted vectors:

$$K = M^{N_t} \qquad (16)$$

with M representing the number of constellation points. So, the complexity grows exponentially with N_t, which is the main disadvantage of this method. For a small number of TX antennas (N_t < 5), however, the complexity is comparable with the other algorithms, described above [11]. Note that for MLD, it is not required that $N_t \leq N_r$.

V. Simulation Results

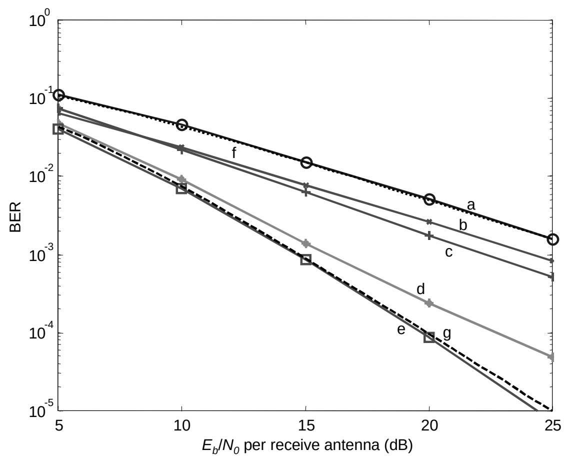

Figure 2: BER versus mean E_b/N_0 per RX antenna for N_t=2 and N_r=2, BPSK, no coding, and a) ZF, b) MMSE, c) ZF with DFB, d) MMSE with DFB, e) MLD, f) theoretical curve of ZF and g) upperbound of MLD [11].

The above described SDM techniques are programmed in Matlab and simulations are performed to compare the BER performances. In Figure 2 the BERs for the different SDM techniques are depicted against E_b/N_0 per receiving antenna for an antenna configuration of $(N_t,N_r) = (2,2)$. Furthermore, a BPSK modulation scheme is used and the data is transmitted without coding.

From Figure 2 it can be seen that MLD has the best performance. According to the BER analysis given in Subsection IV.A, the diversity order of the ZF algorithm for this system is equal to $N_r – N_t+1 = 2–2+1 = 1$. The theoretical curve for ZF (see Formula (8)) is given by the dotted line (curve f) and shows the correctness of the ZF simulation (curve a).

Furthermore, from Figure 2 it can be seen that the MLD curve (curve e) has a diversity of 2 whereas the slope of the other methods tends to a curve with a first order of diversity. In order to check the diversity for the MLD method, some simulations are performed for different antenna configurations. The results are shown in Figure 3.

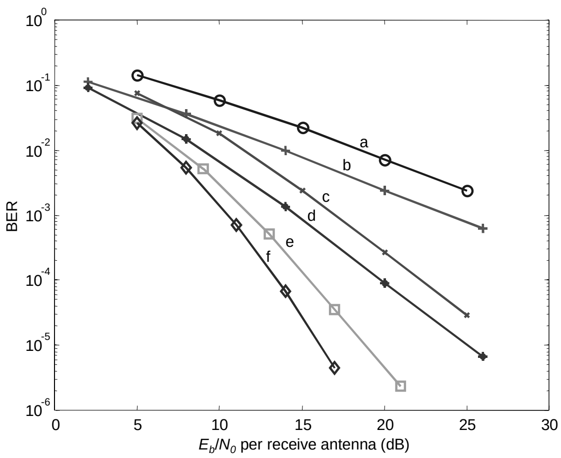

Figure 3: BER versus mean E_b/N_0 per receiving antenna for MLD, BPSK, no coding and for antenna configuration (N_t,N_r) equal to a) (2,1), b) (1,1), c) (3,2), d) (2,2), e) (3,3) and f) (4,4).

From Figure 3 it can be seen that the diversity order of a system based on the MLD technique is equal to the number of receive antennas (N_r) [12]. So the diversity orders of the curves a, b, c, d, e and f are, respectively, 1, 1, 2, 2, 3 and 4. The most interesting conclusion is that, by introducing an extra transmit *and* an extra receive antenna, the BER performance *and* the capacity increase! Finally, it should be noted that the BER performance of a MLD system does not lose its diversity order if the number of transmitting antennas is increased, but the overall performance deteriorates (i.e., the curves of Figure 3 shift to the right when more transmit antennas are added).

VI. APPLICATIONS

Currently, research is ongoing to combine single-carrier narrowband MLD with the spectrum efficient multi-carrier transmission technique Orthogonal Frequency Division Multiplexing [13], to obtain even higher bit-rates.

VII. CONCLUSIONS

Space Division Multiplexing, especially the Maximum Likelihood Decoding (MLD) algorithm proposed in this paper, is a promising solution to achieve high spectrum efficiencies in richly scattered environments. Because SDM exploits the multipath scattering properly, the system capacity *and* the Bit Error Rate (BER) performance are improved. The MLD algorithm appears to have the best BER performance. From simulations, it is concluded that its diversity order, in case of Rayleigh fading, is equal to the number of receive antennas, whereas, the diversity order of the described Zero Forcing algorithm equals the number of receive antennas minus the number of transmit antennas plus one. The main disadvantage of MLD is that its complexity grows exponentially with the number of transmit antennas (N_t), however, for small N_t, the complexity is manageable [11].

REFERENCES

[1] G.J. Foschini and M.J. Gans, "On Limits of Wireless Communications in a Fading Environment When Using Multiple Antennas", *Wireless Personal Communications*, Volume 6, no. 3, pp. 311-335, March 1998.

[2] G.G. Raleigh and J.M. Cioffi, "Spatio-temporal coding for wireless communication", *IEEE Trans. on Communications*, vol. 46, no. 3, pp. 357-366, March 1998.

[3] J.H. Winters, J. Salz and R.D. Gitlin, "The inpact of antenna diversity on the capacity of wireless communication systems", *IEEE Trans. on Commun.*, vol. 42, no. 2, pp. 1740-1751, Feb./Mar./Apr. 1994.

[4] W.C. Jakes Jr., *Microwave Mobile Communications*, John Wiley and Sons, New York, 1974, Chapter 1.

[5] P.W. Wolniansky, G.J. Foschini, G.D. Golden and R.A. Valenzuela, "V-Blast: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel", *1998 URSI International Symposium on Signals, Systems, and Electronics, ISSSE 98*, Pisa, pp. 295-300, 29 Sept. - 2 Oct. 1998.

[6] D. Agrawal, V. Tarokh, A. Naguib and N. Seshadri, "Space-time coded OFDM for high data-rate wireless communication over wideband channels", *48th IEEE Vehicular Technology Conference 1998*, vol. 3, pp. 2232-2236, 1998.

[7] A.F. Naguib, V. Tarokh, N. Seshadri and A.R. Calderbank, "A Space-Time Coding Modem for High-Data-Rate Wireless Communications", *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1459 - 1478, Oct. 1999.

[8] J.G. Proakis, *Digital Communications*, Third Edition, McGraw-Hill, New York, 1995, McGraw-Hill Series in Electrical and Computer Engineering.

[9] G. Strang, *Linear Algebra and its Applications*, Third Edition, Harcourt Brace Jovanovich, Publishers, San Diego, 1988.

[10] T. Kailath, A.H. Sayed and B. Hassibi, *State Space Estimation*, Prentice-Hall, New Jersey, 1999.

[11] A. van Zelst, "Extending the Capacity of Next Generation Wireless LANs Using Space Division Multiplexing combined with OFDM", Eindhoven University of Technology, Faculty of Electrical Engineering, Telecommunication Technology and Electro-magnetics, 1999, Thesis.

[12] R. van Nee, A. van Zelst and G.A. Awater, "Maximum Likelihood Decoding in a Space Division Multiplex System", *IEEE VTC 2000-Spring*, Japan, May 2000.

[13] A. van Zelst, R. van Nee and G.A. Awater, "Space Division Multiplexing (SDM) for OFDM systems", *IEEE VTC 2000-Spring*, Japan, May 2000.