A Compact Representation of Spatial Correlation in MIMO Radio Channels

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***Abstract*—Spatial fading correlation is a crucial impairment for practical Multiple-Input Multiple-Output (MIMO) wireless communication systems. Hence, in system simulations spatial correlation should be taken into account. The main disadvantage, however, is that in general it is represented by a large number of parameters, namely, the various correlation matrix entries. In this paper, we introduce a compact representation of the spatial correlation having at most two coefficients, which nevertheless results in exactly the same capacity and Bit Error Rate (BER) performance. Moreover, this exact mapping allows one to perform MIMO system simulations with spatial correlation, while it is not required to explicitly specify the antenna array design and propagation environment to include correlation.**

Keywords—MIMO systems; spatial fading correlation.

I. INTRODUCTION

In wireless communications, MIMO techniques have recently emerged as a new paradigm to achieve very high bandwidth efficiencies [1]. The concept is based on using multiple transmit and multiple receive antennas along with proper MIMO encoding and detection algorithms ([1, 2]). The spectral efficiency that can be exploited depends strongly on the multidimensional statistical behavior of the MIMO fading channel, partly characterized by the spatial fading correlation.

Spatial correlation in the MIMO context has attracted a lot of attention in literature [3-9]. To our knowledge, up to this moment, when MIMO system simulations are performed, the spatial correlation is included explicitly by means of measured correlation matrices or based on ray tracing. This approach is cumbersome and has as major disadvantage that the MIMO channel statistics are represented by a large number of parameters, namely, the various correlation matrix entries and, therefore, it is hard to cover a wide range of best-case to worst-case scenarios. In this paper, we introduce a compact mapping of the spatial correlation to at most two coefficients, which nevertheless results in exactly the same capacity and BER performance. This simplified correlation model can be used in narrowband simulations directly or in wideband contexts representing the MIMO fading sub-processes per delay tap.

II. MIMO SIGNAL MODEL

Consider a MIMO system with N_t transmit (TX) and N_r receive (RX) antennas. The transmitter emits an $N_t \times 1$ complex signal vector **s**. The receiver records an $N_r \times 1$ complex vector **x**,

$$\mathbf{x} = \mathbf{Hs} + \mathbf{n}\,, \tag{1}$$

where **H** is an $N_r \times N_t$ complex propagation matrix. Element (q,p) of **H** contains the flat-fading channel coefficient from TX antenna p to RX antenna q with variance $\sigma_c^2 = 1$. The vector **n** represents zero mean, complex Additive White Gaussian Noise (AWGN) with covariance matrix $E[\mathbf{nn}^H] = \sigma_n^2\mathbf{I}$, where $(.)^H$ denotes the conjugate transpose of the corresponding vector or matrix and **I** represents the identity matrix (here, $N_r \times N_r$). The total transmit power is $E[\mathbf{s}^H\mathbf{s}] = N_t\sigma_s^2$ and set to P. The SNR per RX antenna equals $\rho = N_t\sigma_s^2/\sigma_n^2$.

III. SPATIAL CORRELATION MODEL

First, we will describe the general spatial correlation definitions and then we will derive a compact mapping of the spatial correlation having at most two coefficients.

In [3], the spatial fading correlation for a narrowband flat-fading MIMO channel **H** is defined as

$$\mathbf{R}_H = E[\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^H]\,, \tag{2}$$

where vec(**H**) denotes the $N_rN_t \times 1$ vector composed by stacking the columns of **H**. In richly-scattered environments, the spatial correlation between the transmit antennas ($\mathbf{R}_{\mathrm{TX}}$) can be assumed to be independent from the correlation between the receive antennas ($\mathbf{R}_{\mathrm{RX}}$) [6], therefore $\mathbf{R}_H$ can be written as [3-6]

$$\mathbf{R}_H = \mathbf{R}_{\mathrm{TX}}^T \otimes \mathbf{R}_{\mathrm{RX}}\,, \tag{3}$$

with $\otimes$ denoting the Kronecker product. Furthermore, $(.)^T$ stands for the transpose of the corresponding matrix, and $\mathbf{R}_{\mathrm{TX}}$ and $\mathbf{R}_{\mathrm{RX}}$ are defined as

$$\mathbf{R}_{\mathrm{TX}} = E[(\mathbf{h}^q)^H\mathbf{h}^q]\,,\ \text{for all } q = 1,\ldots,N_r\,, \tag{4}$$

This work is sponsored by Agere Systems, the Netherlands, and the Dutch cooperative research project B4 BroadbandRadio@Hand, BTS01063.

$$\mathbf{R}_{\mathrm{RX}} = E[\mathbf{h}_p \mathbf{h}_p^H], \text{ for all } p = 1,\dots,N_t, \tag{5}$$

where $\mathbf{h}^q$ is the q-th row of $\mathbf{H}$, and $\mathbf{h}_p$ is the p-th column of $\mathbf{H}$.

To generate independent narrowband flat-fading MIMO channel realizations with spatial correlation, the following expression can be used ([4])

$$\mathbf{H} = \mathrm{unvec}\left(\mathbf{R}_H^{\frac{1}{2}} \mathbf{g}\right), \tag{6}$$

where $\mathbf{g}$ is an $N_t N_r \times 1$ stochastic vector with i.i.d. zero-mean unit variance complex Gaussian elements, and unvec(.) is the reverse of the vec(.) operation. By using some special properties of $\mathbf{R}_H$ and a Kronecker product identity, we can write (6) in a more commonly used form. Note that $\mathbf{R}_H$ is Hermitian and nonnegative definite. Hence, we may write

$$\begin{aligned}\mathbf{R}_H &= \mathbf{U}^H \mathbf{\Lambda}_H \mathbf{U} = \mathbf{U}^H \mathbf{\Lambda}_H^{\frac{1}{2}} \mathbf{\Lambda}_H^{\frac{1}{2}} \mathbf{U} \\ &= \left(\mathbf{\Lambda}_H^{\frac{1}{2}} \mathbf{U}\right)^H \mathbf{\Lambda}_H^{\frac{1}{2}} \mathbf{U} = \left(\mathbf{R}_H^{\frac{1}{2}}\right)^H \mathbf{R}_H^{\frac{1}{2}},\end{aligned} \tag{7}$$

from which we obtain the "square-root" of $\mathbf{R}_H$. Such decompositions also hold for $\mathbf{R}_{\mathrm{TX}}$ and $\mathbf{R}_{\mathrm{RX}}$, so from (3) it follows that

$$\begin{aligned}\mathbf{R}_H &= \mathbf{R}_{\mathrm{TX}}^T \otimes \mathbf{R}_{\mathrm{RX}} \\ &= \left[\left(\mathbf{R}_{\mathrm{TX}}^{\frac{1}{2}}\right)^H \mathbf{R}_{\mathrm{TX}}^{\frac{1}{2}}\right]^T \otimes \left[\left(\mathbf{R}_{\mathrm{RX}}^{\frac{1}{2}}\right)^H \mathbf{R}_{\mathrm{RX}}^{\frac{1}{2}}\right] \\ &= \left\{\left(\mathbf{R}_{\mathrm{TX}}^{\frac{1}{2}}\right)^* \otimes \mathbf{R}_{\mathrm{RX}}^{\frac{1}{2}}\right\}^H \left\{\left(\mathbf{R}_{\mathrm{TX}}^{\frac{1}{2}}\right)^* \otimes \mathbf{R}_{\mathrm{RX}}^{\frac{1}{2}}\right\} \\ &\equiv \left(\mathbf{R}_H^{\frac{1}{2}}\right)^H \mathbf{R}_H^{\frac{1}{2}},\end{aligned} \tag{8}$$

where * denotes the point-wise conjugation. Furthermore, the property is used that, for any matrix $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ with proper dimensions, $(\mathbf{AB}) \otimes (\mathbf{CD}) = (\mathbf{A} \otimes \mathbf{C})(\mathbf{B} \otimes \mathbf{D})$. Based on the Kronecker product identity that for any (complex) $M \times N$ matrix $\mathbf{A}$, $N \times P$ matrix $\mathbf{B}$, and $P \times Q$ matrix $\mathbf{C}$, $\mathrm{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\mathrm{vec}(\mathbf{B})$, (6) can be rewritten as

$$\begin{aligned}\mathbf{H} &= \mathrm{unvec}\left(\mathbf{R}_H^{\frac{1}{2}} \mathbf{g}\right) \\ &= \mathrm{unvec}\left(\left\{\left[\left(\mathbf{R}_{\mathrm{TX}}^{\frac{1}{2}}\right)^H\right]^T \otimes \mathbf{R}_{\mathrm{RX}}^{\frac{1}{2}}\right\}\mathrm{vec}(\mathbf{G})\right) \\ &= \mathbf{R}_{\mathrm{RX}}^{\frac{1}{2}} \mathbf{G} \left(\mathbf{R}_{\mathrm{TX}}^{\frac{1}{2}}\right)^H,\end{aligned} \tag{9}$$

where $\mathbf{G} = \mathrm{unvec}(\mathbf{g})$ is a stochastic $N_r \times N_t$ matrix with i.i.d. complex Gaussian zero-mean unit variance elements. This result is equivalent to the correlation model introduced in [7].

When system simulations need to be carried out, one way to proceed is to explicitly state specific correlation matrices $\mathbf{R}_{\mathrm{TX}}$ and $\mathbf{R}_{\mathrm{RX}}$ covering various propagation scenarios. To obtain these specific correlation matrices, either ray tracing or correlation measurements have to be performed representing different scenarios. This approach is cumbersome and has as major disadvantage that the MIMO fading correlation statistics are represented by a large number of parameters, namely, the various correlation matrix entries and, therefore, it is hard to cover a wide range of best-case to worst-case scenarios. This leads to the question how to reduce this amount of parameters.

We start with the observation that the capacity and BER performance are frequently used measures to evaluate MIMO systems. In the next sections, we introduce a compact representation of the spatial correlation that nevertheless results in an equivalent capacity and BER performance.

IV. Mapping of the Spatial Correlation with Respect to Capacity

The capacity of an $N_r \times N_t$ narrowband MIMO channel $\mathbf{H}$ is given by ([1])

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H\right) \text{ bits/s/Hz.} \tag{10}$$

When spatial correlation is present, the capacity equals

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{R}_{\mathrm{RX}}^{\frac{1}{2}}\mathbf{G}\left(\mathbf{R}_{\mathrm{TX}}^{\frac{1}{2}}\right)^H \mathbf{R}_{\mathrm{TX}}^{\frac{1}{2}}\mathbf{G}^H\left(\mathbf{R}_{\mathrm{RX}}^{\frac{1}{2}}\right)^H\right). \tag{11}$$

With the equality $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$, this can be rewritten to

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{G}\mathbf{R}_{\mathrm{TX}}\mathbf{G}^H\mathbf{R}_{\mathrm{RX}}\right) \text{ bits/s/Hz.} \tag{12}$$

For high SNRs, we get

$$\begin{aligned}C &\approx \log_2 \det\left(\frac{\rho}{N_t}\mathbf{G}\mathbf{R}_{\mathrm{TX}}\mathbf{G}^H\mathbf{R}_{\mathrm{RX}}\right) \\ &= \log_2 \det\left(\frac{\rho}{N_t}\mathbf{I}_{N_r}\right)\det(\mathbf{G})\det(\mathbf{R}_{\mathrm{TX}})\det\left(\mathbf{G}^H\right)\det(\mathbf{R}_{\mathrm{RX}}),\end{aligned} \tag{13}$$

since the determinant of a product is the product of the determinants. So, apparently, the capacity distributions of two different situations will be the same when the determinant of the $\mathbf{R}_{TX}$'s and $\mathbf{R}_{RX}$'s are equal. Or, it must be possible to introduce a model that is, in a capacity sense, a mapping of measured correlation matrices. To that end, we require

$$\det(\mathbf{R}_{\mathrm{TX,mod}}) = \det(\mathbf{R}_{\mathrm{TX,meas}}), \quad (14)$$

$$\det(\mathbf{R}_{\mathrm{RX,mod}}) = \det(\mathbf{R}_{\mathrm{RX,meas}}). \quad (15)$$

Note that in case the correlation matrices of a possible model would be set equal on both sides of the communication link, i.e., $\mathbf{R}_{\mathrm{TX,mod}} = \mathbf{R}_{\mathrm{RX,mod}} = \mathbf{R}_{\mathrm{mod}}$, we would get the criterion

$$\det(\mathbf{R}_{\mathrm{mod}}) = \sqrt{\det(\mathbf{R}_{\mathrm{TX,meas}})\ \det(\mathbf{R}_{\mathrm{RX,meas}})}. \quad (16)$$

Now the question is if there exists a unique solution for the requirements (14) and (15). To answer that question, note that, like $\mathbf{R}_H$, both $\mathbf{R}_{TX}$ and $\mathbf{R}_{RX}$ are nonnegative definite. According to Hadamard's inequality for an $N \times N$ nonnegative definite matrix $\mathbf{A}$ ([10]),

$$\det(\mathbf{A}) \leq \prod_{i=1}^{N} a_{ii}, \quad (17)$$

where a_{ii} represents the i-th diagonal element of $\mathbf{A}$. For the correlation matrices $\mathbf{R}_{TX}$ and $\mathbf{R}_{RX}$ this means that $\det(\mathbf{R}_{TX}) \leq 1$ and $\det(\mathbf{R}_{RX}) \leq 1$. Furthermore, since the determinant of a matrix is the product of the eigenvalues and since the eigenvalues of a nonnegative definite matrix are real and nonnegative, this yields $\det(\mathbf{R}_{TX}) \geq 0$ and $\det(\mathbf{R}_{RX}) \geq 0$. So, the determinant of the (measured) correlation matrices will always be real, larger than or equal to zero and less than or equal to one.

Next, it is shown that a unique match can be found with respect to capacity using the following simple and generic definitions for the transmitter and receiver correlation[1]:

$$\mathbf{R}_{\mathrm{TX,mod}} = \begin{pmatrix} 1 & r_{TX} & r_{TX}^2 & \cdots & r_{TX}^{N_t-1} \\ r_{TX} & 1 & r_{TX} & \ddots & \vdots \\ r_{TX}^2 & r_{TX} & 1 & \ddots & r_{TX}^2 \\ \vdots & \ddots & \ddots & \ddots & r_{TX} \\ r_{TX}^{N_t-1} & \cdots & r_{TX}^2 & r_{TX} & 1 \end{pmatrix}, \quad (18)$$

$$\mathbf{R}_{\mathrm{RX,mod}} = \begin{pmatrix} 1 & r_{RX} & r_{RX}^2 & \cdots & r_{RX}^{N_r-1} \\ r_{RX} & 1 & r_{RX} & \ddots & \vdots \\ r_{RX}^2 & r_{RX} & 1 & \ddots & r_{RX}^2 \\ \vdots & \ddots & \ddots & \ddots & r_{RX} \\ r_{RX}^{N_r-1} & \cdots & r_{RX}^2 & r_{RX} & 1 \end{pmatrix}, \quad (19)$$

[1] Note that a similar model has been introduced in [9] with the difference that in [9] the correlation is defined as $\mathbf{R} = E[\mathbf{H}\mathbf{H}^H]$.

where r_{TX} and r_{RX} represent (real-valued) correlation coefficients. The most powerful property of this model is that, when ranging the coefficients between 0.0 and 1.0, we can in a controlled way go from fully uncorrelated scenarios (all off-diagonal elements of both matrices equal to 0.0) to fully correlated scenarios (all entries equal to 1.0). Another useful property is the simple form of the determinants of these matrices. The determinant of, e.g., $\mathbf{R}_{\mathrm{TX,mod}}$ can be shown to be

$$\det(\mathbf{R}_{\mathrm{TX,mod}}) = \left(1 - r_{TX}^2\right)^{N_t-1}. \quad (20)$$

Finally, it can be shown that the determinant for the *modeled* matrices is monotonically decreasing in the range of interest, e.g., $\mathbf{R}_{\mathrm{TX,mod}}$ as function of r_{TX} is monotonically decreasing for $0 \leq r_{TX} \leq 1$ (see Figure 1 for N_t is 2, 3 and 4). Based on these observations, it can be concluded that there will always be a unique mapping that satisfies the criteria (14) and (15).

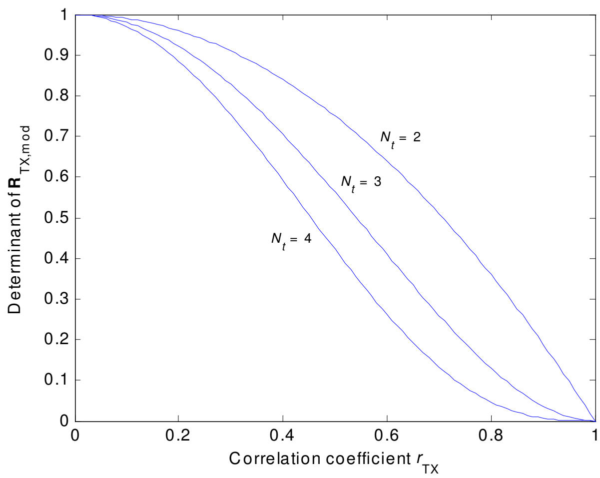

Figure 1: The determinant of the correlation model matrix $R_{\mathrm{TX,mod}}$ versus the correlation coefficient r_{TX} for a various number of TX antennas.

Since a mathematical link is found to match the MIMO capacity of measured correlation matrices with that of the model, we can suffice with one example. The result is presented for complex correlation matrices measured in a picocell environment ([4]) and given by (21) and (22).

For these measured matrices, it can be shown that $\det(\mathbf{R}_{\mathrm{TX,meas}}) = 0.2372$ and $\det(\mathbf{R}_{\mathrm{RX,meas}}) = 0.2796$, respectively. From the criteria (14), (15), $0 \leq r_{TX} \leq 1$, and $0 \leq r_{RX} \leq 1$, we obtain $r_{TX} = 0.6172$ and $r_{RX} = 0.5883$. With these results, the capacity of the measured correlation matrices can be compared with that of the model. Note that for every realization of $\mathbf{G}$, (11) produces a different instantaneous capacity value. The average of these capacity values, i.e., the ergodic capacity, as function of the average SNR per receive antenna is shown in Figure 2 for the measured and modeled correlation matrices. From these curves, we indeed see that the match is perfect, even for low SNR values.

$$\mathbf{R}_{\text{TX,meas}} = \begin{pmatrix} 1 & -0.45+0.53i & 0.37-0.22i & 0.19+0.21i \\ -0.45-0.53i & 1 & -0.35-0.02i & 0.02-0.27i \\ 0.37+0.22i & -0.35+0.02i & 1 & -0.10+0.54i \\ 0.19-0.21i & 0.02+0.27i & -0.10-0.54i & 1 \end{pmatrix}. \quad (21)$$

$$\mathbf{R}_{\text{RX,meas}} = \begin{pmatrix} 1 & -0.13-0.62i & -0.49+0.23i & 0.15+0.28i \\ -0.13+0.62i & 1 & -0.13-0.52i & -0.38+0.12i \\ -0.49-0.23i & -0.13+0.52i & 1 & 0.02-0.61i \\ 0.15-0.28i & -0.38-0.12i & 0.02+0.61i & 1 \end{pmatrix}. \quad (22)$$

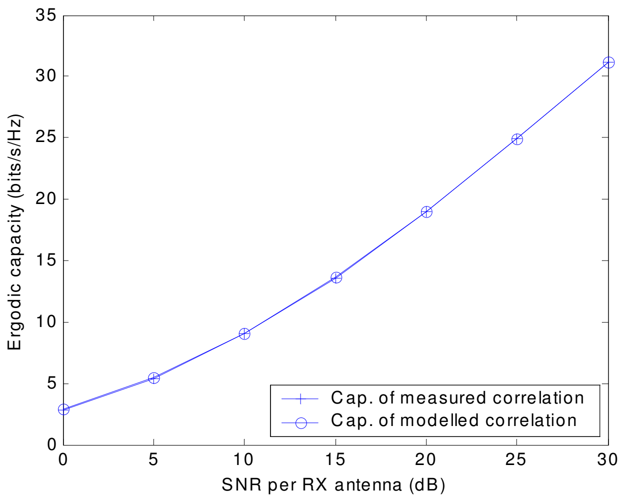

Figure 2: Ergodic capacity versus SNR per RX antenna for measured and modelled spatial correlation for a 4×4 system.

Obviously, the introduced spatial correlation model may not be an accurate model for some real-world scenarios, but it is a simple dual-coefficient model that allows one to study the effect of correlation on the MIMO capacity in an explicit way. Moreover, with the criteria (14) and (15), we found a simple mapping with measured correlation matrices. In the next section, a compact representation of the spatial correlation in BER performance evaluations is obtained.

V. Mapping of the Spatial Correlation with Respect to the BER Performance

In this paper, Maximum Likelihood Detection (MLD) [2] is selected as MIMO detection scheme to find a compact representation of the spatial correlation in BER performance evaluations. To that end, we will use the Pairwise Error Probability (PEP) as a performance measure. Let $\mathbf{s}_i$ and $\mathbf{s}_k$ be two possible spatial TX vectors with dimensions $N_t \times 1$ and assume that $\mathbf{s}_i$ is transmitted. Then, with $\mathbf{y} = \mathbf{H}(\mathbf{s}'_i - \mathbf{s}'_k)$, where $\mathbf{s}'_i$ and $\mathbf{s}'_k$ are the normalized versions of $\mathbf{s}_i$ and $\mathbf{s}_k$, respectively, such that $\mathbf{s}'_i = \mathbf{s}_i/\sigma_s$ and $\mathbf{s}'_k = \mathbf{s}_k/\sigma_s$, and using the same approach as in [11], the PEP of MLD can be shown to be [12]

$$\Pr(\mathbf{s}_i \to \mathbf{s}_k) \le \det\left(\mathbf{I}_{N_r} + \frac{\sigma_s^2}{4\sigma_n^2}\mathbf{Q}_y\right)^{-1}, \quad (23)$$

where $\mathbf{Q}_y$ is the covariance matrix of $\mathbf{y}$. For a high SNR the PEP can be approximated by

$$\Pr(\mathbf{s}_i \to \mathbf{s}_k) \le \det\left(\frac{\rho}{4N_t}\mathbf{Q}_y\right)^{-1}. \quad (24)$$

Hence, in the asymptotic case, the PEP (and thus the BER performance) depends inversely on the determinant of $\mathbf{Q}_y$. Now the question arises: what is $\mathbf{Q}_y$ in scenarios with spatial correlation? To find the answer, we start by rewriting $\mathbf{y}$:

$$\mathbf{y} = \mathbf{H}(\mathbf{s}'_i - \mathbf{s}'_k) = \left((\mathbf{s}'_i - \mathbf{s}'_k)^T \otimes \mathbf{I}_{N_r}\right)\mathrm{vec}(\mathbf{H}). \quad (25)$$

Note that when $\mathbf{s}'_i$ and $\mathbf{s}'_k$ have a mean of zero, $\mathbf{y}$ is also zero mean. Now it can be shown that, by averaging over $\mathbf{H}$, the covariance matrix of $\mathbf{y}$ equals

$$\begin{aligned} \mathbf{Q}_y &= E[\mathbf{y}\mathbf{y}^H] \\ &= \left((\mathbf{s}'_i - \mathbf{s}'_k)^T \otimes \mathbf{I}_{N_r}\right)\mathbf{R}_H\left((\mathbf{s}'_i - \mathbf{s}'_k)^* \otimes \mathbf{I}_{N_r}\right) \\ &= \left((\mathbf{s}'_i - \mathbf{s}'_k)^T \otimes \mathbf{I}_{N_r}\right)\left(\mathbf{R}_{\text{TX}}^T \otimes \mathbf{R}_{\text{RX}}\right)\left((\mathbf{s}'_i - \mathbf{s}'_k)^* \otimes \mathbf{I}_{N_r}\right) \\ &= (\mathbf{s}'_i - \mathbf{s}'_k)^T \mathbf{R}_{\text{TX}}^T (\mathbf{s}'_i - \mathbf{s}'_k)^* \otimes \mathbf{R}_{\text{RX}} = \beta\mathbf{R}_{\text{RX}}. \end{aligned} \quad (26)$$

From this result, we can observe that, in order to have the same PEP for the modeled and measured $\mathbf{R}_{\text{RX}}$, the determinants of both matrices must be the same; $\det(\mathbf{R}_{\text{RX,mod}}) = \det(\mathbf{R}_{\text{RX,meas}})$. And by using (19) we can deduce an r_{RX} that achieves an equivalent MLD performance compared to the performance with the measured spatial receiver correlation $\mathbf{R}_{\text{RX,meas}}$.

Regarding the spatial correlation at the transmitter, it is obvious that β strongly depends on $(\mathbf{s}_i - \mathbf{s}_k)$. Therefore, to find a link between $\mathbf{R}_{\text{TX,mod}}$ and $\mathbf{R}_{\text{TX,meas}}$, one has to average over all possible difference vectors $(\mathbf{s}_i - \mathbf{s}_k)$, which is equivalent to using the overall error rate performance. An upperbound on the overall error rate performance can be found by averaging over all PEP's by means of, e.g., the union bound. Since the $\mathbf{s}_i$'s are taken from a discrete set that depends on the constellation size, the easiest and most effective way to find a link is through numerical evaluation.

Because we found a (numerical) mathematical mapping between the MLD error rate performance for measured and modeled spatial correlation matrices, one example showing the match is sufficient. To that end, we will again use the measured spatial correlation matrices as given by (21) and (22). Clearly, the matching criterion of the measured and modeled spatial correlation at the receiver side for the MLD error rate performance is equivalent to (15). So to link (19) with (22), r_{RX} must be set to 0.6172. Furthermore, from numerical evaluation we found that r_{TX} must be set to 0.38. Finally, the match is shown graphically in Figure 3 in which a perfect match of the upperbounds can be observed. The slight mismatch between the simulation curves at high SNR can be explained mainly by their limited accuracy. The curves are namely obtained by averaging over 100,000 64-byte packets.

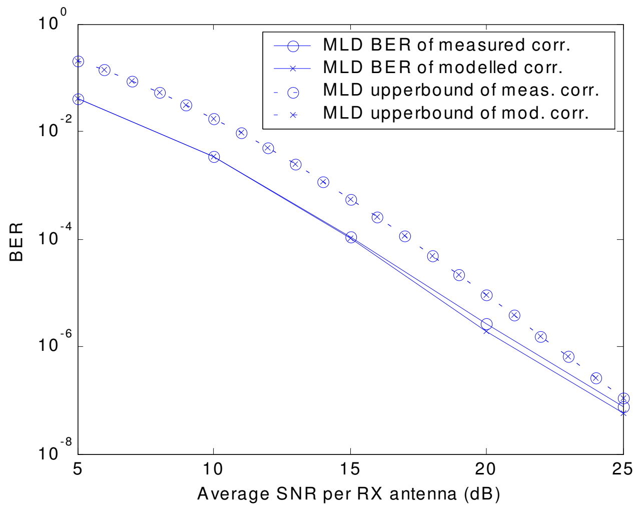

Figure 3: MLD BER performance and upperbound versus average SNR per RX antenna for measured and modelled spatial correlation for a 4 × 4 system.

VI. CORRELATION DELAY PROFILE

The spatial correlation model described in this paper is a narrowband model. The maximum of two parameters, however, allow us to easily extend this to a wideband channel model. In general the correlation changes over the time interval of the wideband channel impulse response. These variations can be captured in what is referred to as the Correlation Delay Profile (CDP). One can imagine that in real-world environments, the first channel taps are mainly determined by a few strong paths, e.g., the LOS path and some dominant reflections, whereas towards the last taps of the channel impulse response, the angular spread is more omni-directional with many (equally strong) contributing paths. This results in a high correlation coefficient at the beginning of the CDP and low values at the end.

VII. CONCLUSIONS

We have introduced a simple representation of spatial correlation in MIMO radio channels. For the frequently used evaluation measures of a MIMO system, namely capacity and BER performance, the amount of parameters representing the spatial correlation can be reduced to at most two. With a proper choice of these coefficients, the correlation can be varied controllably from the totally uncorrelated scenario to the fully correlated scenario. This simplified correlation model allows one to perform simulations with spatial correlation, while it is not required to explicitly specify the hardware (e.g. antenna) setup and wave propagation environment to include the spatial correlation. Altogether, this makes the model powerful, yet simple to use.

ACKNOWLEDGEMENT

The author would like to thank J.S. Hammerschmidt for the valuable discussions.

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