# A SINGLE COEFFICIENT SPATIAL CORRELATION MODEL FOR MULTIPLE-INPUT MULTIPLE-OUTPUT (MIMO) RADIO CHANNELS

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## ABSTRACT

Spatial fading correlation is one of the impairments practical Multiple-Input Multiple-Output (MIMO) wireless communication systems have to cope with. The designer of such systems should take into account that the system performance can substantially degrade when correlation is present. Therefore, we introduce an easy-to-use single parameter MIMO spatial correlation model that nevertheless reflects the relevant characteristics of the real-life propagation phenomena appropriately. We demonstrate how an excellent match between the BER performance based on measurements and that based on the introduced model can be achieved, as well as its flexibility to model a wide range of propagation environments.

## 1 INTRODUCTION

In wireless communications, Multiple-Input Multiple-Output (MIMO) techniques have recently emerged as a new paradigm. It has been shown that MIMO is a promising approach that can lead to very large bandwidth efficiencies [1]. This spectral efficiency is achieved by transmitting different signals on different transmit antennas simultaneously in the same frequency channel as well as using multiple receive antennas and appropriate receive schemes. ([1-4]).

The spectral efficiency that can be exploited in MIMO systems depends on a number of phenomena, including the average received power of the desired signal, thermal and implementation-related noise, as well as co-channel interference. Moreover, the multidimensional statistical behaviour of the MIMO fading channel is of foremost significance to the system performance. Therefore, it is crucial for the designer of a MIMO communication system to have a MIMO spatial correlation model that appropriately reflects the essential propagation characteristics.

Spatial fading correlation in the MIMO context has recently attracted a lot of attention in literature [5-11] and various signal models that include spatial correlation have been proposed. To our knowledge, however, literature is still lacking a simple but accurate correlation model that can be varied from an uncorrelated to a fully correlated scenario. The goal of this paper is to introduce a simple single parameter correlation model for MIMO systems. We will restrict ourselves to the flat-fading (narrowband) case. Note, however, that a wideband model can easily be constructed from independent multiple flat-fading sub-processes based on the reasonable assumption of uncorrelated scattering (US) (see [12]).

## 2 MIMO CHANNEL MODEL

A point-to-point communication system comprising $N_t$ transmit (TX) and $N_r$ receive (RX) antennas is considered. The transmitter sends an $N_t$-dimensional complex signal vector $\mathbf{s}$. The receiver records an $N_r$-dimensional complex vector $\mathbf{x}$, which depends on $\mathbf{s}$ via

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n}\,, \tag{1}$$

where $\mathbf{H}$ is an $N_r \times N_t$ complex propagation matrix that is constant per packet transmission (i.e., quasi-static) and assumed to be known at the receiver (e.g., using training sequences). Element $(n,m)$ of $\mathbf{H}$ contains the normalised flat-fading channel response from TX antenna $m$ to RX antenna $n$ with variance $\sigma_c^2 = 1$. The vector $\mathbf{n}$ is $N_r$-dimensional and represents zero mean, complex Additive White Gaussian Noise (AWGN) with covariance matrix $E[\mathbf{n}\mathbf{n}^H] = \sigma_n^2\mathbf{I}$, where $(.)^H$ denotes the conjugate transpose of the corresponding vector or matrix and $\mathbf{I}$ represents the identity matrix, here having dimension $N_r$. The total power of $\mathbf{s}$ (i.e., $E[\mathbf{s}^H\mathbf{s}] = N_t\sigma_s^2$) is $P$, i.e., independent of the number of TX antennas.

## 3 SPATIAL CORRELATION MODEL

In [5-8], the correlation between different MIMO channel elements is modelled under the assumption that the correlation among receive antennas is independent of the correlation between transmit antennas (and vice versa). The underlying justification for this approach is to assume that only immediate surroundings of the antenna array impose the correlation between array elements and have no impact on correlations observed between the elements of the array at the other end of the link, which is a reasonable assumption for indoor environments [9]. A way to include this type of antenna signal correlation into the MIMO channel model for Rayleigh flat-fading like channels, is given by

$$\mathbf{H} = \mathbf{R}_{H,\mathrm{RX}}^{\frac{1}{2}} \mathbf{G} \mathbf{R}_{H,\mathrm{TX}}^{\frac{1}{2}}, \tag{2}$$

where $\mathbf{G}$ is a stochastic $N_r \times N_t$ matrix with independent, identically distributed (i.i.d.) complex Gaussian zero-mean unit variance elements. $\mathbf{R}_{H,\mathrm{TX}}$ ($N_t \times N_t$ dimensional) and $\mathbf{R}_{H,\mathrm{RX}}$ ($N_r \times N_r$ dimensional) denote the correlation observed on the transmitter and receiver side, respectively. With $\mathbf{h}^n$ denoting the $n$-th row of $\mathbf{H}$ and $\mathbf{h}_m$ the $m$-th column of $\mathbf{H}$, these correlation matrices can be found by $\mathbf{R}_{H,\mathrm{TX}} = E[(\mathbf{h}^n)^H \mathbf{h}^n]$, for $n = 1,\ldots,N_r$ and $\mathbf{R}_{H,\mathrm{RX}} = E[\mathbf{h}_m (\mathbf{h}_m)^H]$, for $m = 1,\ldots,N_t$.

When simulations need to be carried out, one way to proceed is to explicitly use a large amount of measured data for $\mathbf{R}_{H,\mathrm{TX}}$ and $\mathbf{R}_{H,\mathrm{RX}}$ covering target environments in the channel model simulation. This approach has the main disadvantage that the essential properties of the channel (does it have favourable or unfavourable MIMO statistics?) are concealed in a larger number of parameters, i.e., the various entries for the correlation matrices. Hence, a detailed analysis is always required to complement a given set of data. We are, therefore, interested in a more "transparent" modelling approach. Such a model should (1) reflect real-life MIMO channel statistics according to the target radio environment and system parameters, (2) cover a wide range of best-case to worst-case scenarios, and (3) be easy to use and have the possibility to convey channel model parameters between various groups of researchers. To satisfy these requirements, we will use the approximation for the fading correlation between two adjacent antenna elements averaged over all possible orientations of the two antennas in a given wave-field [10]:

$$r(d) \approx \exp\left(-23\Lambda^2 d^2\right). \tag{3}$$

In this equation, $d$ is the distance in wavelengths between two antennas and $\Lambda$ is the angular spread according to [10]. Note that this definition of $\Lambda$ is very general, i.e., it is defined for any distribution of power in the azimuth plain, and values close to 0.0 denote completely directional scenarios whereas those at 1.0 represent more uniform spreading of energies in space. Based on this approximation, a correlation model is proposed for linear arrays at both the transmitter and the receiver with equidistant antenna spacings $d_{\mathrm{TX}}$ and $d_{\mathrm{RX}}$, respectively, at the TX and RX side of the communication link, resulting in the following Toeplitz structure correlation matrices:

$$\mathbf{R}_{H,\mathrm{TX}} = \begin{bmatrix} 1 & r_{\mathrm{TX}} & r_{\mathrm{TX}}^4 & \cdots & r_{\mathrm{TX}}^{(N_t-1)^2} \\ r_{\mathrm{TX}} & 1 & r_{\mathrm{TX}} & \ddots & \vdots \\ r_{\mathrm{TX}}^4 & r_{\mathrm{TX}} & 1 & \ddots & r_{\mathrm{TX}}^4 \\ \vdots & \ddots & \ddots & \ddots & r_{\mathrm{TX}} \\ r_{\mathrm{TX}}^{(N_t-1)^2} & \cdots & r_{\mathrm{TX}}^4 & r_{\mathrm{TX}} & 1 \end{bmatrix} \text{ and } \mathbf{R}_{H,\mathrm{RX}} = \begin{bmatrix} 1 & r_{\mathrm{RX}} & r_{\mathrm{RX}}^4 & \cdots & r_{\mathrm{RX}}^{(N_r-1)^2} \\ r_{\mathrm{RX}} & 1 & r_{\mathrm{RX}} & \ddots & \vdots \\ r_{\mathrm{RX}}^4 & r_{\mathrm{RX}} & 1 & \ddots & r_{\mathrm{RX}}^4 \\ \vdots & \ddots & \ddots & \ddots & r_{\mathrm{RX}} \\ r_{\mathrm{RX}}^{(N_r-1)^2} & \cdots & r_{\mathrm{RX}}^4 & r_{\mathrm{RX}} & 1 \end{bmatrix}, \tag{4}$$

where $r_{\mathrm{TX}}$ and $r_{\mathrm{RX}}$ represent $r(d_{\mathrm{TX}})$ and $r(d_{\mathrm{TX}})$, respectively. Note that this model can range from the totally uncorrelated scenario ($r_{\mathrm{TX}} = r_{\mathrm{RX}} = 0$) to the fully correlated scenario ($r_{\mathrm{TX}} = r_{\mathrm{RX}} = 1$). For small $r_{\mathrm{TX}}$ and $r_{\mathrm{RX}}$ (much smaller than 1), we can discard the higher order terms in (4), which translates the Toeplitz matrices to a tridiagonal structure. This tridiagonal correlation model can be viewed as approximating a scenario in which it is assumed that the fading correlation has a certain given value for any pair of adjacent antenna elements, and that any other pair of antenna elements exhibits independent fading, on both sides of the communication link. For further simplification of the spatial correlation, we can set $r_{\mathrm{TX}} = r_{\mathrm{RX}} = r$, which leads to a single-parameter MIMO correlation model. In order to verify this modelling approach, the following section carries out a sanity check of the model from various perspectives.

## 4 VERIFICATION OF THE SPATIAL CORRELATION MODEL

### 4.1 Validation with Respect to the Capacity

$$C = \log_2 \det\left( \mathbf{I}_{N_r} + \frac{\rho}{N_t} \mathbf{H}\mathbf{H}^H \right), \tag{5}$$

represents the generalised Shannon bound on the capacity for MIMO systems [1], where $\rho$ is the average Signal-to-Noise Ratio (SNR) per RX antenna given by $N_t\sigma_s^2/\sigma_n^2$. Note that every realisation of **H**, defined by (2), produces a different instantaneous capacity value. The average of these capacity values (i.e., the ergodic capacity) is given in Fig. 1 for the Toeplitz and tridiagonal correlation models together with the capacity for uncorrelated (full rank) and fully correlated (rank-1) fading scenarios. Clearly, the tridiagonal correlation model is only valid for small $r$, whereas the Toeplitz model ranges from the fully uncorrelated to the totally correlated scenario by simply changing $r$ from 0 to 1.

### 4.2 Bit Error Rate (BER) Comparison between the Modelled and Measured Correlation Matrices

In [7, 8] measured correlation matrices for a 4 × 4 system have been derived. We have carried out the BER evaluation for all these measured matrices and found excellent matches between the model and the measured data for a careful choice of the correlation parameter. Here, we will work out environment FB7B2 [8] as an example. For this environment the correlation matrices are given by

$$\mathbf{R}_{H,\mathrm{TX}} = \begin{bmatrix} 1 & 0.3169 & 0.3863 & 0.0838 \\ 0.3169 & 1 & 0.7128 & 0.5626 \\ 0.3863 & 0.7128 & 1 & 0.5354 \\ 0.0838 & 0.5626 & 0.5354 & 1 \end{bmatrix} \text{ and } \mathbf{R}_{H,\mathrm{RX}} = \begin{bmatrix} 1 & 0.1317 & 0.1992 & 0.2315 \\ 0.1317 & 1 & 0.1493 & 0.1907 \\ 0.1992 & 0.1493 & 1 & 0.1996 \\ 0.2315 & 0.1907 & 0.1996 & 1 \end{bmatrix}. \tag{6}$$

Now, let $\mathbf{H}_{\mathrm{FB7B2}}$ and $\mathbf{H}_m$ denote a realisation of **H** based on these matrices and on the Toeplitz model, respectively. One way to compare the spatial correlation model with measured data is to perform the eigenvalue analysis of the instantaneous covariance matrix $\mathbf{Q} = \mathbf{H}\mathbf{H}^H$ , like in [8]. Note that a channel matrix **H** effectively offers up to $K$ parallel subchannels of different strengths $\lambda_k$ ($1 \le k \le K$), with $K = \mathrm{rank}(\mathbf{Q}) \le \min(N_t,N_r)$ [8]. The joint distribution of the square roots of these eigenvalues ($\lambda_k$) represents exactly the distribution of **H**, as shown in [3]. For visualisation reasons, we will, however, show the cumulative distribution functions (cdfs) of the eigenvalues, keeping in mind that a match between these cdfs for the modelled and measured correlation matrices does not necessarily mean that the distribution of $\mathbf{H}_{\mathrm{FB7B2}}$ and $\mathbf{H}_m$ are completely identical. For a match in the BER performance of MIMO Zero Forcing (ZF) detection [4], curves like Fig. 2 have been used to estimate a suitable $r$. On a trial and error basis we found that the match of the BER curves is excellent for $r$ = 0.42 (see Fig. 3). After more evaluations we found that to have an agreement in BER performance of ZF, the cdfs of the lowest eigenvalue should match (see Fig. 4). This can be explained by the fact that the lowest eigenvalue represents the lowest subchannel gain. Thus, this subchannel has the lowest SNR, resulting for ZF in the largest contribution to the BER. For MIMO Maximum Likelihood Detection (MLD) [4], however, the BER performance based on $\mathbf{H}_{\mathrm{FB7B2}}$ and $\mathbf{H}_m$ is in better agreement for $r$ = 0.5 (see Fig. 3). This can be explained by the fact that for MLD, a match in BER performance is achieved when the cdfs of the highest eigenvalues agree. These results show that if a scan is carried out for $r$ values ranging from best-case ($r$ = 0.0) to worst-case ($r$ = 1.0) conditions, the simulations will also cover the equivalent of the FB7B2 scenario – for $r$ being in the region of 0.4 - 0.5. Note that the above measured data are in fact for a non-uniform antenna array structure; correspondingly, the Toeplitz correlation model can be used to capture best and worst case scenarios for any other given array structures including 3D setups, use of polarization diversity, antenna pattern shaping or other elements available in the design space.

## 5 CONCLUSIONS

We have introduced a single coefficient spatial correlation model for MIMO radio channels. We have shown that an excellent agreement in BER performance can be found between the model and measured data. Furthermore, the strength of the model is that the spatial correlation can be varied from the totally uncorrelated scenario to the fully correlated scenario – without explicitly specifying the hardware setup as well as the wave propagation environment. Altogether, this makes the model powerful, yet simple to use.

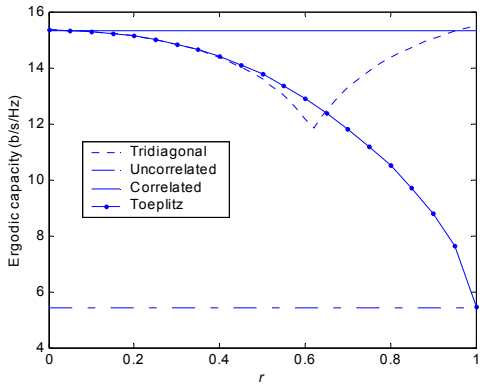


Fig. 1: Ergodic $C$ vs. $r$ for different spatial correlation models, for a $4 \times 4$ system and an SNR of 20 dB.

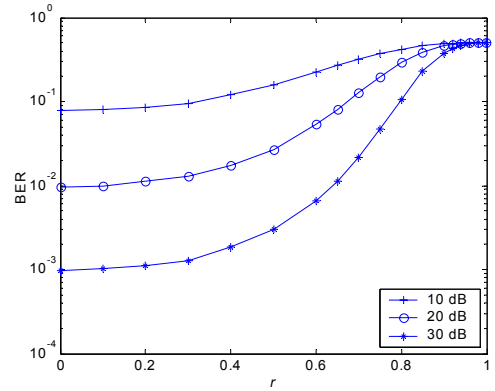


Fig. 2: BER of single carrier $4 \times 4$ MIMO using ZF vs. $r$ for different SNRs per RX antenna and BPSK.

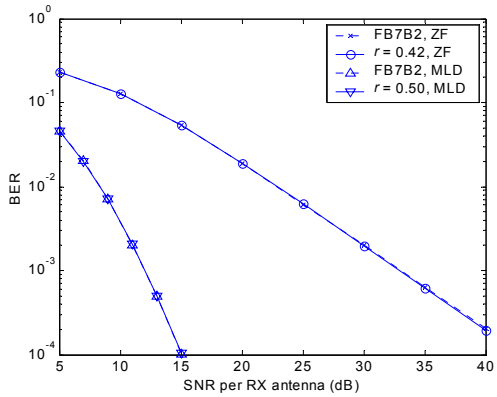


Fig. 3: BER vs. SNR per RX antenna for $4 \times 4$ MIMO with ZF or MLD, BPSK, and measured (dotted lines) or Toeplitz correlation matrices (solid lines).

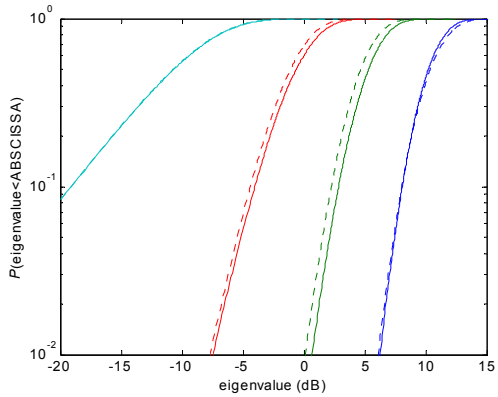


Fig. 4: cdfs of ordered eigenvalues for simulated (solid lines, $r = 0.42$) and measured data (dotted lines) for the FB7B2 environment.

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