Space Division Multiplexing (SDM) for OFDM systems

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Abstract - A promising solution for significant increase of the bandwidth efficiency and transmission capacity is the exploitation of the spatial dimension, by using Space Division Multiplexing (SDM). SDM algorithms exploit the richly scattered (indoor) wireless channel by using multiple transmit and receive antennas. In this paper, a new SDM technique, called Maximum Likelihood Decoding (MLD) is proposed. The superior SNR performance of MLD compared to other SDM techniques is proven. To obtain an even higher bit rate and make the system more robust against Inter Symbol Interference, (single-carrier) SDM is successfully applied to the spectrum efficient multi-carrier transmission technique Orthogonal Frequency Division Multiplexing (OFDM).

I. Introduction

The main goals in developing new wireless communication systems are increasing the transmission capacity (or bit rates) and improving the spectrum efficiency. A promising solution for significant increase of the bandwidth efficiency and performance under noise is the exploitation of the spatial dimension. Recent information theory research has revealed that the (indoor) multipath wireless channel is capable of enormous capacities, provided that the multipath scattering is sufficiently rich ([1-3]). Solutions that exploit the multipath properly can be captured under the term Space Division Multiplexing (SDM) or Space Division Multiple Access (SDMA). Basically, these techniques transmit different data streams on different transmit antennas simultaneously, with the goal of increasing the capacity and the Signal-to-Noise Ratio (SNR) performance. When using multiple antennas at the receiver as well, these different data streams, that are mixed-up in the air, can be recovered by SDM techniques like Zero Forcing (ZF) [3] or V-BLAST (Vertical Bell laboratories Layered Space Time) [4]. With a prototype of BLAST, bandwidth efficiencies of 20 – 40 bps/Hz have been demonstrated in an indoor environment at realistic SNRs and error rates. In this paper another SDM algorithm is introduced, called Maximum Likelihood Decoding (MLD). It is shown to have the best performance. Furthermore, MLD is combined with Orthogonal Frequency Division Multiplexing (OFDM) [5] to avoid Inter Symbol Interference (ISI) and to make the system more robust against frequency selective fading.

Note that the difference between SDM and SDMA is, that the latter allows different users to transmit simultaneously on a single antenna each, whereas in SDM a single user transmits simultaneously on multiple antennas.

Currently, a lot of research is ongoing to apply transmitter or receiver diversity to multi-carrier techniques like OFDM. A number of transmitter and receiver diversity techniques for OFDM are proposed in [6, 7] and [8-10], respectively. The difference with SDM, is that with the latter the SNR performance *and* data rate can be improved.

Closely related to the SDM techniques are the so-called Space-Time codes [11, 12]. The difference is that for Space-Time codes, the data is coded in the space (on different antennas) and time dimension to maximize the coding gain and diversity gain. Thus, one data-symbol is coded and all antennas are used to transmit it. SDM achieves higher bit rates by transmitting different data-symbols on the different transmit antennas simultaneously. Coding gain is obtained by encoding the data in advance with a convolutional code.

II. Multi-Antenna Link: Signal Model

In this section, a signal model for the multi-antenna link will be stated in which the communication channel bandwidth is assumed to be so narrow that the channel can be treated as flat with frequency (i.e., flat fading).

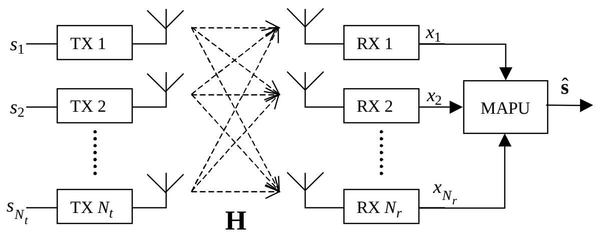

Figure 1: The physical model of a system with SDM. (MAPU = Multi Antenna Processing Unit)

A communication system comprising N_t transmit (TX) and N_r receive (RX) antennas will be considered. It is assumed to operate in a Rayleigh flat-fading environment and exploits the spatial dimension by using Space Division Multiplexing (SDM) (see Figure 1). At discrete times, the transmitter sends an N_t-dimensional (complex) signal vector **s**. The receiver records an N_r-dimensional complex vector **x**. The following signal model describes the relation between **s** and **x**:

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \boldsymbol{\nu} \qquad (1)$$

where **H** is an $N_r \times N_t$ complex propagation matrix that is constant with respect to the symbol time and assumed known at the receiver (e.g. via transmitting training sequences). Since it is assumed that the system operates in a Rayleigh flat-fading environment, it can be said that **H** has independently and identically distributed (i.i.d.), zero-mean, circularly-symmetric, complex Gaussian entries with unit variance (the variance of each entry is $\sigma_c^2 = 1$) [13]. The N_r-dimensional vector **v** represents zero mean, complex Additive White Gaussian Noise (AWGN) with covariance matrix:

$$E[\mathbf{v}\mathbf{v}^*] = \sigma_v^2 \mathbf{I}_{N_r} \tag{2}$$

where * denotes the conjugate transpose of a matrix. The matrix **I** with subscript N_r represents the identity matrix with dimension N_r. The vector **s** is assumed to have zero-mean, uncorrelated random variables with variance σ_s^2. The total power of **s** (i.e., $E[\mathbf{s}^*\mathbf{s}]$) is assumed to be P (independent of the number of transmit antennas!). Thus, the covariance matrix of **s** equals:

$$E\left[\mathbf{s}\mathbf{s}^*\right] = \sigma_s^2 \mathbf{I}_{N_t} = \frac{P}{N_t}\mathbf{I}_{N_t} \tag{3}$$

Furthermore, the vectors **s** and **v** are assumed to be independent ($E[\mathbf{s}\,\mathbf{v}^*]=0$). Now, the expected Signal-to-Noise Ratio (SNR) per receiving antenna, i.e. the SNR for each component of **x**, can be found and is equal to:

$$\rho = \frac{E_s}{N_0} = \frac{N_t \sigma_s^2 \sigma_c^2}{\sigma_v^2} = \frac{P}{\sigma_v^2} \tag{4}$$

where E_s stands for the signal power per receive antenna and N_0 denotes the noise power per receive antenna.

III. Capacity

Provided that the channel matrix **H** is known at the receiver, the Shannon capacity for an SDM system is given by ([1]):

$$\begin{aligned} C &= \log_2 \det\left(\mathbf{I}_{N_r} + \frac{1}{\sigma_v^2}\mathbf{H}\mathbf{Q}\mathbf{H}^*\right) \\ &= \log_2 \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^*\right) \text{ bps/Hz} \end{aligned} \tag{5}$$

where **Q** denotes the covariance matrix of the transmitted vector **s** given by Formula (3). As an example, lets consider the capacity for $N_r=N_t=N$ in the limit of large N. By the law of large numbers, $\mathbf{H}\mathbf{H}^*/N \rightarrow \mathbf{I}_N$ almost surely as N gets large, so, the capacity for large N is asymptotic to:

$$C = N \log_2(1+\rho) \text{ bps/Hz} \tag{6}$$

From this formula, it can be concluded that for high SNRs and $N_r=N_t=N$, the scaling of the capacity is like N more bps/Hz for every 3 dB SNR improvement, whereas, for a conventional wireless communication system ($N_t=N_r=1$) with the same total transmission power P, the scaling is only 1 bps/Hz [1].

IV. Maximum Likelihood Decoding

The SDM technique described in this paper is called Maximum Likelihood Decoding (MLD). In MLD, **s** is estimated according to the Maximum Likelihood principle. The idea is to find a vector $\mathbf{s}_j$ for which the probability $P(\mathbf{s}_j|\mathbf{x})$ is maximized (with $1 \le j \le K$), where K denotes all possible transmitted vectors:

$$K = M^{N_t} \tag{7}$$

with M representing the number of constellation points. Using Bayes' rule, this probability may be expressed as:

$$P\left(\mathbf{s}_j \middle| \mathbf{x}\right) = \frac{p\left(\mathbf{x}\middle|\mathbf{s}_j\right)P\left(\mathbf{s}_j\right)}{p(\mathbf{x})} \tag{8}$$

where $p(\mathbf{x}|\mathbf{s}_j)$ is the conditional probability density function (pdf) of the observed vector, given that $\mathbf{s}_j$ is transmitted. $P(\mathbf{s}_j)$ is the probability of the j-th vector being transmitted. If the K vectors are equally probable to be transmitted, then $P(\mathbf{s}_j) = 1/K$. Furthermore, the denominator in Formula (8) is independent of $\mathbf{s}_j$. Consequently, finding the vector that maximizes $P(\mathbf{s}_j|\mathbf{x})$ is equivalent to finding the vector that maximizes $p(\mathbf{x}|\mathbf{s}_j)$.

The (conditional) pdf $p(\mathbf{x}|\mathbf{s}_j)$ is a complex multivariate normal distribution. The general formula of a complex multivariate normal distribution **x**, with mean **μ** and covariance matrix **Q**, can be shown to be [14]:

$$p(\mathbf{x}) = \det(\pi\mathbf{Q})^{-1}\exp\left(-(\mathbf{x}-\boldsymbol{\mu})^*\mathbf{Q}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right) \tag{9}$$

For a specific channel **H** and given $\mathbf{s}_j$, this leads to:

$$p\left(\mathbf{x}\middle|\mathbf{H},\mathbf{s}_j\right) = \det(\pi\mathbf{Q})^{-1}e^{-\left(\mathbf{x}-\mathbf{H}\mathbf{s}_j\right)^*\mathbf{Q}^{-1}\left(\mathbf{x}-\mathbf{H}\mathbf{s}_j\right)} \tag{10}$$

With **Q** equal to:

$$\begin{aligned} \mathbf{Q} &= E[(\mathbf{x}-\boldsymbol{\mu})(\mathbf{x}-\boldsymbol{\mu})^*] \\ &= E\left[\left(\mathbf{x}-\mathbf{H}\mathbf{s}_j\right)\left(\mathbf{x}-\mathbf{H}\mathbf{s}_j\right)^*\right] = E\left[\mathbf{v}\mathbf{v}^*\right] = \sigma_v^2\mathbf{I}_{N_r} \end{aligned} \tag{11}$$

this results in the conditional pdf:

$$p\left(\mathbf{x}\middle|\mathbf{H},\mathbf{s}_j\right) = \det(\pi\mathbf{Q})^{-1}e^{-\frac{1}{\sigma_v^2}\left(\mathbf{x}-\mathbf{H}\mathbf{s}_j\right)^*\left(\mathbf{x}-\mathbf{H}\mathbf{s}_j\right)} \tag{12}$$

Consequently, finding the maximum of the conditional probability $P(\mathbf{s}_j|\mathbf{x})$ leads to:

$$\hat{\mathbf{s}} = \arg \max_{\mathbf{s}_j \in \{\mathbf{s}_1,\ldots,\mathbf{s}_K\}} p\left(\mathbf{x} \middle| \mathbf{H}, \mathbf{s}_j\right) = \arg \min_{\mathbf{s}_j \in \{\mathbf{s}_1,\ldots,\mathbf{s}_K\}} \left\|\mathbf{x} - \mathbf{H}\mathbf{s}_j\right\|^2 \quad (13)$$

The last formula is the MLD solution. Note that MLD is optimal in performance, because finding the maximum of the conditional probability $P(\mathbf{s}_j|\mathbf{x})$ leads to the minimization of the symbol error probability [13].

Note that the MLD solution requires an exhaustive search through all possible transmitted vectors K. So, the complexity is proportional to K, which is the main disadvantage of this method. For a small number of transmit antennas (N_t<5), however, the complexity seems reasonable. Note that for ZF and V-BLAST the following must hold: $N_t \leq N_r$ [3, 4], whereas, this is not required for MLD.

V. Simulation Results

The SDM technique MLD is programmed in Matlab and some simulations are performed to obtain the BER performance. In Figure 2, the BERs for the different antenna configurations are depicted against SNR per receiving antenna. Furthermore, a BPSK modulation scheme is used and the data is transmitted without coding.

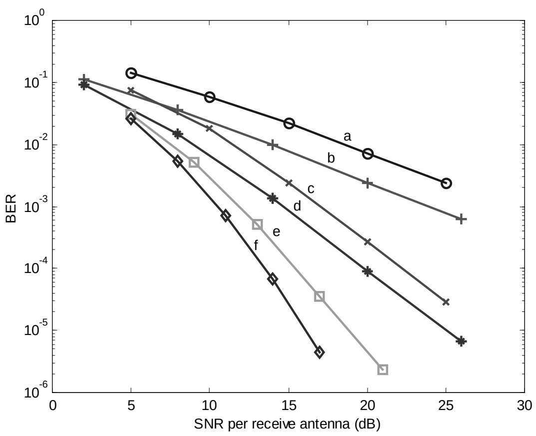

Figure 2: BER versus mean SNR per receiving antenna for MLD, BPSK, no coding and for antenna configuration (N_t,N_r) equal to a) (2,1), b) (1,1), c) (3,2), d) (2,2), e) (3,3) and f) (4,4).

From Figure 2 it can be seen that the diversity order[1] of a system based on the MLD technique is equal to the number of receive antennas (N_r) (see the proof in [15]). So the diversity orders of the curves a, b, c, d, e and f are, respectively, 1, 1, 2, 2, 3 and 4. In [3], it is shown that the ZF algorithm achieves a diversity of N_r–N_t+1, thus, MLD performs significantly better!

The most interesting about MLD is that, by introducing an extra transmit *and* an extra receive antenna, the BER performance *and* the capacity increase! Furthermore, it should be noted that the BER performance of a MLD system does not lose its diversity order if the number of transmitting antennas is increased, but the overall performance deteriorates (i.e., the curves of Figure 2 shift to the right when more transmit antennas are added).

VI. SDM combined with OFDM

In the nearby future, applications that operate on carrier frequencies in the order of several Giga-Hertz, will be based on multicarrier systems, like OFDM [5], as becomes clear from, e.g., the new standards for wireless LANs (for example, the IEEE 802.11a standard) [16]. Therefore, a system that combines SDM with OFDM is proposed in this section. It is assumed that the subchannel bandwidth is so narrow that the communication channel per subcarrier can be treated as flat with frequency.

The SDM algorithms, are single carrier algorithms, so, in order to combine SDM with OFDM, SDM has to be performed for each subcarrier. For a system with antenna configuration (N_t,N_r), every subcarrier bears N_t data streams. At the N_r receivers, the subcarrier information is separated by using Fast Fourier Transformation blocks (FFTs). Then, the N_r information symbols belonging to subcarrier i are routed to the i-th Multi-Antenna Processing Unit (MAPU) where MLD is implemented to recover the N_t transmitted data signals per subcarrier. Finally, demapping, deinterleaving and decoding are performed (see Figure 3).

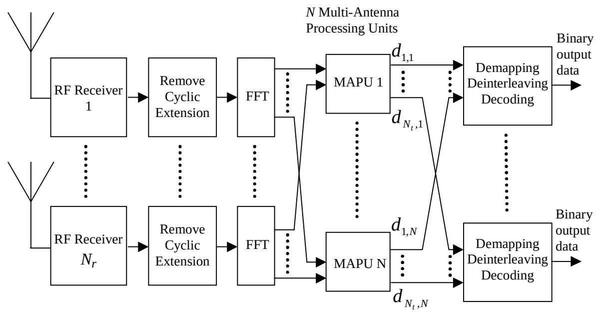

Figure 3: Multi-antenna receiver using SDM combined with OFDM.

The transmitting part can consist of different users (in the case of SDMA) or of a multi-antenna transmitter (in the case of SDM) or a combination of both, that is why the transmitting chains should be separable. This is represented schematically in Figure 4. Note that in case of SDMA, in order to perform the MLD algorithm properly, the signals at the receiver need to be synchronized. Therefore, synchronization of the transmitters is required.

[1] A diversity order of i means that the BER decreases 10^i times if the SNR increases by 10 dB.

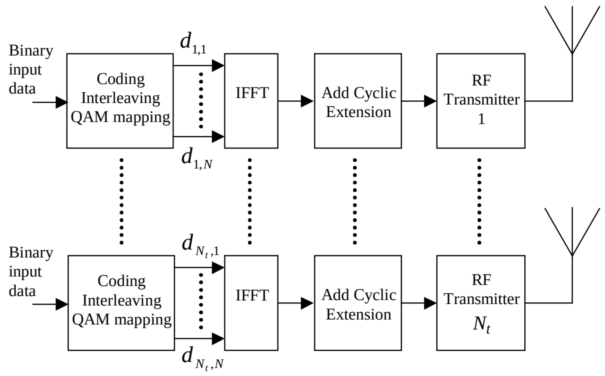

Figure 4: Multi-antenna transmitter(s) using OFDM.

VII. Simulations and Results

A. Simulation parameters

The system proposed in Figure 3 and Figure 4 has been simulated in C++. Table I lists the main simulation parameters. They are based on the OFDM standard IEEE 802.11a [16]. A key parameter is the guard-time interval (T_G) of 800 ns. This interval is introduced to provide robustness to Root Mean Square (RMS) delay spreads (τ_{rms}) up to several hundreds of nanoseconds. In practice, this means that the system is robust enough to be used in any indoor environment, including large factory buildings [16].

In the IEEE 802.11 standard, 48 data subcarriers are used and uncoded data rates of 12 to 72 Mbps can be achieved by using variable modulation types from BPSK to 64-QAM. In order to correct for subcarriers in deep fades, forward error correction across the subcarriers is used with variable coding rates, giving coded data rates from 6 up to 54 Mbps. Convolutional coding is implemented with the industry standard rate 1/2, constraint length 7 code with generator polynomials (133,171).

Table I: Main simulation parameters based on the OFDM standard in IEEE 802.11.

Modulation	Quadrature Phase Shift Keying (QPSK)
Coding rate	1/2
Number of subcarriers	64
Number of subcarriers used	48
OFDM symbol duration	4 µs
Guard interval	800 ns

B. Simulation and results

Simulations are performed for different antenna configurations and/or different delay spreads. Furthermore, it is assumed that every channel element (i.e., the channel between a specific transmit and receive antenna) consists of Rayleigh fading paths with an exponentially decaying power delay profile [16]. Figure 5 shows the Packet Error Rate (PER) versus the mean E_b/N_0 per receive antenna for different delay spreads. The simulation is performed for a (2,2) system (i.e., a system with antenna configuration (N_t,N_r) = (2,2)) and the data is coded with a rate 1/2 convolutional code and T_G = 800 ns. The channel bit rate is 48 Mbit/s, or after coding 24 Mbit/s.

From Figure 5 it can be concluded that at first the PER performance increases when the delay spread increases, but if the delay spread exceeds 267 ns, the performance starts to deteriorate again. This can be explained as follows. If the delay spread increases, the fading changes from flat to frequency selective fading. As a result, there can be several fades within the OFDM signal bandwidth, with relatively strong subcarriers in between. The coding benefits from these stronger subcarriers to compensate for the attenuated subcarriers. Finally, if the delay spread becomes larger than 267 ns, the performance goes down, because in this case the paths with a large delay cannot be resolved with the guard time and will appear as Inter-Symbol Interference (ISI).

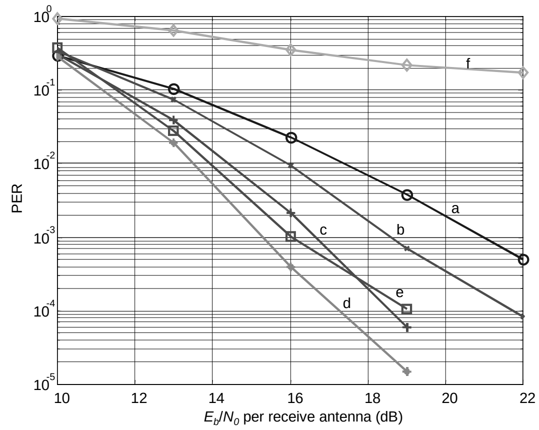

Figure 5: PER versus mean E_b/N_0 per receive antenna for Rayleigh fading paths with an exponentially decaying power delay profile, QPSK, 64 byte packets, rate 1/2 coding, (N_t,N_r) = (2,2). RMS delay spread is a) 17, b) 33, c) 67, d) 133, e) 267, f) 533 ns.

The resulting ISI creates an irreducible error floor, even for high SNRs. Thus, it might be interesting to look at the PER floor versus the normalized delay spread for different antenna configurations. The results are shown in Figure 6. The simulations are performed with a high SNR value of 80 dB. When comparing curves b, c and d, it can be concluded that an OFDM system becomes more robust against delay spread by introducing extra receiving antennas. This is based on the fact that a system with SDM becomes more robust against Gaussian noise in case of introducing an extra receive antenna (i.e., the order of diversity increases). Apparently, the delayed OFDM symbols have more or less the same characteristics as Gaussian noise.

If an extra transmit antenna is introduced, however, the performance under delay spread deteriorates, which can be explained by the fact that an extra "interferer" is introduced. From Figure 6, however, it can be noticed that a (1,1) system (curve b) and a (2,2) system (curve e) have comparable performance. So, increasing the capacity (by introducing an extra transmit and receive antenna) does not have a negative influence on the delay spread performance!

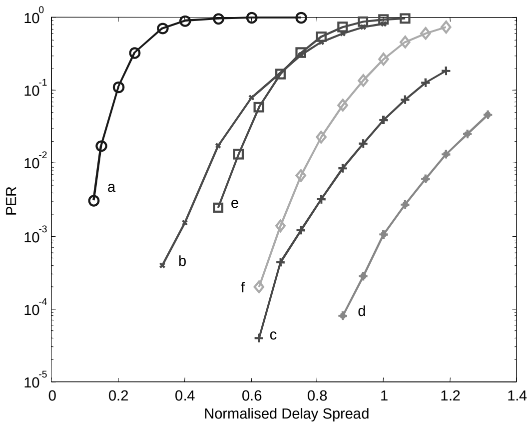

Figure 6: Irreducible packet error ratios versus normalized delay spread τ_{rms}/T_G for 64 byte packets, QPSK, rate 1/2 code (except for curve a) and antenna configuration (N_t,N_r): a) (1,1), no coding, b) (1,1), c) (1,2), d) (1,3), e) (2,2), f) (2,3).

VIII. Conclusions

Space Division Multiplexing, especially the Maximum Likelihood Decoding algorithm proposed in this paper, is a promising solution for achieving spectrum efficient transmission of data in a richly scattered environment. Because SDM exploits the multipath scattering properly, the system capacity *and* the Bit Error Rate (BER) performance can be improved. It is proven that MLD has the best performance and from simulations, it is concluded that its diversity order, in case of Rayleigh fading, is equal to the number of receive antennas.

MLD is successfully applied to OFDM to achieve more robustness against frequency selective fading. The delay spread channel is modeled by exponentially-decayed Rayleigh fading. From simulation results, it is concluded that the delay spread tolerance does not deteriorate by introducing multiple transmit and receive antennas.

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